A

TREATISE

OF

Practical Arithmetic,

BOTH

Integral *and* Fractional.

WITH

The MENSURATION of all Sorts of Bodies, both *Superficially* and *Solidly*.

The Whole after a New Method.

Accommodated to the Capacity of Beginners.

By *ALEXANDER WRIGHT*, M. A.
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Where may be had,
A TREATISE of FRACTIONS, by the ſame Author.

Place this next the Title.

To the Right Honourable

WILLIAM CHALMERS Esq.r L.d Provoſt.

Alexander Robertſon
Alex.r Mitchel of Colpna
James Moorison Junr.
Mr. William Fordyce
} Baillies.

Willm. Mouat Junr. Dean of Gild.

Andrew Logie, Theaſurer.

And to the Remanent Members of
yr Town Council of ABERDEEN.

The following Treatise
is most humbly Inscribed
by their much Obliged &
most Obedient Servant
Alexander Wright.

Felicissimæ Indolis

Optimæque Spei

Adolescenti,

D. Andreæ Skene

de Lethenty

Hunc Arithmetices Practicæ

Tractatum

Amoris et Observantiæ ergo

D. D. C. Q.

Alexander Wright.

Several denominations in avoirdupois weight not noticed in this treatise

a firkin of butter	56 lb
——— soap	64
a barrel of anchovies	30
——— soap	256
——— raisins	112
a stone of iron shot, or horsemans weight	14
——— butchers meat	8
a gallon of train oil	9.6 oz
a puncheon of prunes	1120
a truss of straw	36
——— new hay	60
——— old hay	56

Thirty-six trusses are a load

cheese or butter	lb
a clove, or half stone	8
a wey, in Suffolk	32c or 256
——— Essex	42c or 336

Coals

3 bushels ——— 1 Sack

12 Sacks or 36 bushels 1 chaldron

21 Chaldrons ——— 1 Score

NB 3 sacks of coals are always allowed by dealers on an order for five chaldron

THE PREFACE.

THE Books already publiſhed on Arithmetic, are ſo very numerous, that the Public will poſſibly be ſurprized to ſee a New One on that Subject; eſpecially when it is conſider'd that by the late Improvements this Science has received, there can little be ſaid on it, but what has been advanced by ſome one Author or other. It would ſeem therefore that all that can be expected now, is to ſee the moſt eaſy, plain, ſhort, and familiar Method of inſtructing Youth in this excellent Art, to which I flatter myſelf I have (at leaſt) paved the Way in the following Sheets; a Specimen whereof I ſhall here briefly lay before the Reader. The Method is very much new. In the Integral Part I have retrench'd the Number of what they call the Rules, and have reduced them all (after the firſt five general ones) to three, viz. the Rule of Three Numbers, Rule of Five Numbers, and Rules of Practice, comprehending in the firſt of theſe, thoſe that are commonly called the Rules of Intereſt, Diſcount, Tare and Trett, Fellowſhip, Gain and Loſs, Alligation, Barter and Exchange. The Examples adduced are choice, very practical, and contain a vaſt Variety, each of which I have illuſtrated; ſo that there can remain no Difficulty in underſtanding the Method and Reaſon of operating. In the Fractional Part, which is ſo ſuperficially handled by moſt Authors, I have been very copious and plain, that the Learner may not be diſcouraged in proſecuting ſo uſeful and neceſſary a Part of the Science, and have placed them immediately after Integral Diviſion, as judging it proper they ſhould be learned before one

commenceth the Rule of Three; *becauſe their Uſe and Application moſtly appears there, and in what follows afterwards. To the End of each* Rule *I have ſubjoined ſome uſeful* Practical Queſtions, *ſhewing its immediate Application to Buſineſs. The* Appendix *contains a further Explication of* Intereſt, *both* Simple *and* Compound, *with all the* Tables *that are neceſſary for that purpoſe; as alſo a fuller and plainer Account of the Menſuration of all Sorts of Bodies, than is commonly to be met with in Treatiſes of Arithmetic. So that a Learner may, without the Aſſiſtance of a Teacher,* ſooner *attain to a competent Knowledge of Numbers, by ſtudying them in the Method here laid down, than any other I have had the opportunity of ſeeing. In the* Decimal Part, *I have purpoſely omitted* Infinites, Circulates, *and* Approximates, *which the Reader may ſee fully explain'd, if he pleaſes to conſult a* Treatiſe of Fractions *lately publiſhed by me, to which I refer him.*

——— Si quid noviſti rectiùs iſtis,
Candidus imperti: Si non, his utere mecum.

HORAT.

From my School in the *Upper Kirk-ſtreet, Aberdeen.*

ERRATA.

PAg. 5. for 11, 111, 111, *lege* 12, 101, 111. *P.* 45, in *Exa.* 3. the Diviſor 8 is wanting. *P.* 48. *l.* 19. for $\frac{1}{3}$ *lege* $\frac{11}{3}$. *P.* 58. *l.* 15. for 7 d. *read* 17 d. *p.* 60. *l.* 11. for 15 d. *lege* 15 $\frac{1}{2}$. — *l.* 15. *lege* 11 s. 6 d. — — *lege* 13. s.

INTRODUCTION.

I HAVE thought it proper in this Place, to premiſe the Definitions of ſome Terms, as alſo the Significations of ſome Marks or Symbols, made Uſe of in the following Sheets for Brevity's ſake.

1. A Number is either Unity or Multitude.

2. An Integer, or whole Number, is 1, or any Number of Units.

3. A Fraction is a Part or Parts of Unity, according as the ſame is divided.

4. An Aliquot Part is a leſſer Number, which is contained a certain Number of Times in a greater, without a Remainder; as 4 is an aliquot Part of 8, 12, 16, 20, *&c.*

5. An Aliquant is a leſſer Number, which is contained in a greater a certain Number of Times, with ſomething remaining: Thus 4 is an aliquant Part of 10, 14, 18, *&c.*

6. An Abſtract Number is a Number conſidered abſolutely in itſelf, without any Name or Denomination applied to it.

7. An Applicate Number is ſuch as hath ſome Name applied to it: Thus 6, taken abſolutely, and ſignifying ſimply ſix, is Abſtract; but if we annex to it the word Men, Pounds, Gallons, *&c.* it is Applicate.

8. The Common Meaſure to any two or more Numbers, is a Number which can divide thoſe Numbers, without a Remainder: Thus 6 is a Common Meaſure to 18, 24, 36, *&c.*

9. The Greateſt Common Meaſure to any two or more Numbers, is the greateſt Number which can

divide the propoſed Numbers, without a Remainder: Thus 6 is the greateſt Common Meaſure to 18 and 24; likewiſe 4 is the greateſt Common Meaſure to 8, 12, 20.

10. A Prime Number is that which has no Meaſure but itſelf and Unity: Thus 3, 5, 7, 11, 13, 23, 29, *&c.* are Prime Numbers.

11. The Multiplier and Multiplicand go frequently by the name of Factors, becauſe by being multiplied together they make the Product.

12. When a Sum of Money is lent, or lies out for any Time, it is called a Principal, and the Money paid for the Uſe or Forbearance of the Principal, is commonly called Intereſt, and is always at ſome certain Rate *per Cent. per Annum*, ſuch as 4, 5, 6, *&c.*

13. Simple Intereſt is that which is produced by the Principal lent or forborn for any Time. But,

14. Compound Intereſt is that which is produced by the Principal and the Simple Intereſt lying out unpaid for any Time, both in one Sum. For Example, *L.* 100 lent at 5 *per Cent.* makes at the End of the Year *L.*105; and if this *L.*105 be let lie in the Borrower's hands, and become a new Principal to bear Intereſt the next Year, then it it is called Compound Intereſt: And ſo on for any Number of Years, by ſtill adding the preceding Year's Intereſt to its Principal.

15. The Amount of any Principal for any Time is the Sum of that Principal and all the Intereſts due upon the ſame.

16. When a Sum of Money, payable at any Time hence, and not bearing Intereſt till after it is due, is to be paid preſently, the Creditor muſt allow the Debtor ſo much for advancing it before the Time, as, being put out to Intereſt from this Time to that of the Payment of the Debt, would amount to the Intereſt of the Debt for the ſame time, at any Rate *per Cent.* agreed on: and the Money ſo allowed is called

Diſ-

Diſcount or Rebate; which Diſcount being ſubtracted from the Debt firſt due, the Remainder is the preſent Worth.

17. An Annuity is an annual Payment, or a Sum of Money payable every Year for a certain Time or for ever, whether it be due on Lands, Houſes, or Money in Stock or Bank, *&c.* And when an Annuity lies unpaid for any Time, it is ſaid to be in Arrears.

18. Bartering is the exchanging of one Commodity for another.

19. Tare is an Allowance for the Weight of the Hogſhead, Cask, Cheſt or Bag, *&c.* which contain the Goods, and may either be known ſeparately, from the Weight of the Goods, or is accounted ſo much *per Cent.* or 112 *lib.*

Trett is an Allowance on ſome Sort of Goods for Waſte, Duſt, Refuſe or Inlack, *&c.*

Cloff is an Allowance of 2 *lib.* on every Draught above 3 *C.* to Freemen of *London.*

When the Tare is ſubtracted, the Remainder is called Suttle Weight; and when all Allowances are deducted, what remains is called Nett Weight or Weight payable.

EXPLANATION of the MARKS or SYMBOLS.

The Sign + (*plus* or more) is the Sign of Addition
The Sign — (*minus* or leſs) denotes Subtraction.
The Sign ÷ (divided by) is the Sign of Diviſion.
The Sign × (multiply'd by) is the Sign of Multiplication.
The Sign = (equal to) is the Sign of Equality.

ARITHMETIC

IS the Art of Numbering.

The Characters whereby Numbers are expreſſed, are,

1. 2. 3. 4. 5. 6. 7. 8. 9. 0.

NOTATION

TEacheth to expreſs any Sum or Number by theſe Figures, according to the following

TABLE.

Places.	Names.
1	Units.
2	Tens.
3	Hundred.
4	Thouſand.
5	Ten Thouſ.
6	C Thouſands.
7	Millions.
8	X Millions.
9	C Millions.
10	M Millions.
11	X Th. Mills.
12	C Th. Mills.
13	Billions.

Or thus:

Th. Bills. Bills. Th. Mills. Mills. Thouſ. Units.

C X Un. C X Un. C X Un. C X Un. C X Un. C X Un.

&c.

Read thus: Units, Tens, Hundreds of Units; Units, Tens, Hundreds of Thouſands, *&c.* ſo that when a Sum is propoſed to be read or expreſſed in Words, divide the ſame into Periods, each conſiſting of

of 3 Figures, and read each Period as if it was an entire Number by itſelf, applying always its general Name.

For Examp. Let the Number 14356485 be propoſed to be read or expreſſed in Words; firſt divide it into Periods thus, 14,356,485, and you'll find it conſiſt of 3 Periods, that is, Units, Thouſands Millions, and therefore is read 14 Mill. 3 hund. 56 Thouſand, 4 Hund. eighty five. In like manner,

,758,763,747		758 Mill. 763 Thouſ. 747
12,469,721,766		12 Th. 469 M. 721 Th. 766
4,317,467,005,701	is read	4 B. 317 th. 467 M. 5 th. 701
,400,073,000,465		4 th. 73 Mill. 465.
1,000,000,000,000		1 Billion.

Set down in Figures Four thouſand and eight 4008
Set down Fifty four th. Millions 54,000,000,000
Set down Eleven hundred and eleven 1,111
Set down Elev. hund. and elev. th. and elev. 1,111,011
Set down Eleven hundred and eleven thouſand, one hundred and eleven 1,111,111
Set down Eleven Mill. Eleven hund. thouſ. eleven hund. and eleven 11,111,111

Of the Roman *Notation.*

The *Romans* expreſs'd Numbers by the Capital Letters of their Alphabet, and their ſimple Characters were theſe,

I . V . X . L . C . D . M equal to
1 . 5 . 10 . 50 . 100 . 500 . 1000

And the intermediate Numbers betwixt theſe, they expreſſed by a Repetition of the ſame, ſetting them after one another in a Line, the Characters of the greateſt Value being placed to the left: Thus II=2, VI=6, VIII=8, LX=60, DC=600, DCC=700. But ſometimes they ſet the Character of the leſſer

Value to the left of the greater, and this signified the Difference betwixt the two, which they used for a shorter Expression: Thus IV=4, IIV=3, IX=9, XL=40, CD=400, CM=900; and for D=500 they wrote IↃ, and by adding another Ↄ, it express'd ten Times as much: Thus IↃↃ=5000, IↃↃↃ=50000. Also for M they wrote CIↃ, and by placing C and Ↄ on each hand, it expressed ten Times as much, for CCIↃↃ=10000. They expressed their Thousands sometimes by a Line drawn over the Top of any Number less than 1000, thus, $\overline{V}$=5000, $\overline{X}$=10000.

1 I.
2 II.
3 III.
4 IIII or IV.
5 V.
6 VI.
7 VII.
8 VIII or IIX.
9 VIIII or IX.
10 X.
11 XI.
12 XII.
13 XIII.
18 XVIII or IIXX.
19 XVIIII or XIX.
20 XX.
30 XXX.
40 XXXX or XL.
45 XLV.
49 XLIX.
50 L.
59 LIX.
60 LX.
69 LXIX.
70 LXX.
89 LXXXIX.
100 C.
200 CC.
500 D or IↃ.
600 DC or IↃC.
700 DCC or IↃCC.
1000 M or CIↃ or $\overline{\text{I}}$.
2000 CIↃ CIↃ or $\overline{\text{II}}$.
3000 CIↃ CIↃ CIↃ or $\overline{\text{III}}$.
4000 CIↃ CIↃ CIↃ CIↃ or $\overline{\text{IV}}$.
5000 IↃↃ or $\overline{\text{V}}$.
10000 CCIↃↃ or $\overline{\text{X}}$.
50000 IↃↃↃ or $\overline{\text{L}}$.
100000 CCCIↃↃↃ or $\overline{\text{C}}$.
500000 IↃↃↃↃ.
900 CM.
1000000 CCCCIↃↃↃↃ or $\overline{\text{M}}$.
2000000 $\overline{\text{MM}}$.
1738 { CIↃIↃCCXXXVIII or MDCCXXXIIX.

CHAP.

CHAP. II. *Addition*

Finds the Sum of two or more Numbers propoſed.

I SHALL begin with the Addition of Simple Abſtract or Simple Applicate Numbers, for which obſerve the following

RULE. Set down the ſeveral Numbers under one another, ſo as Units may ſtand under Units, Tens under Tens, *&c.* and, beginning at the right Hand, add together the Figures in each Column ſeverally, and ſet down the Sum, if it does not exceed 9; or, if it is 10 or above, ſet down the Exceſs, carrying 1 for every 10 to the next Column: And thus proceed 'till you come to the laſt Column, ſetting down the Sum of it, becauſe you have no more Columns to add. The Figures thus found are called the Amount, Total, Aggregate or Sum of the ſeveral Particulars given to be added.

Exa. 1.	*Exa.* 2.	*Exa.* 3.
4265	*L.* 3658	*Yds.* 1187
1512	7834	356
Sum 5797	Sum 11492	Sum 1543

Exa. 4.	*Exa.* 5.	*Exa.* 6.
16325	26370	4000
176	285	2000
1483	186	600
37	400	300
218	76	400
105	42	1000
3	27	700
17	112	100
Sum 18364	Sum 27498	Sum 9100

1. *Of*

1. *Of* Engliſh *Money.*

The leaſt Piece of Money now uſed in *England* is a Farthing, whereof 4 make 1 Penny, 12 Pence 1 Shilling, and 20 Shillings 1 Pound: In theſe Accounts are kept, and are marked with the Characters *L. ſ. d. q.* or *f.*

Beſides thoſe they have other Coins, ſome real and others imaginary.

4 Farth.	make	1 Penny.	The real Coins now current, and commonly known, are, *Of Copper,* A Farth. and a Half-penny. *Of Silver,* A Penny, Two-pence, Three-pence, Four-pence, Six-pence, a Shilling, Half a Crown, a Crown.
4 Pence		1 Groat.	
6 Pence		1 Teſter.	
12 Pence		1 Shilling.	
5 Shill.		1 Crown.	
6 Sh. 8 d.		1 Noble.	
10 Shill.		1 Angel.	
13 s. 4 d.		1 Mark.	
20 Shill.		1 Pound.	

And of Gold, Half a Guinea=10 *s.* 6 *d.* and a Guinea=21 Shill. They have alſo ſeveral other Pieces both of Gold and Silver, but they are not ſo common.

In *Scotland* we uſe *L. ſh. d.* and *L.* 12 *Scotch* are equal to 1 *L. Sterling.* But Merchants, and People of Faſhion, generally account in *Engliſh* Money.

Exa.

Exa. 1.

(20)(12)(4)

L.	*s.*	*d. qrs.*
145:	19:	11: 3
26:	8:	6: 1
35:	14:	10: 2
16:	8:	9: 0
43:	17:	10: 3
54:	13:	4: 0
18:	6:	8: 2
20:	13:	9: 1
Sum 362:	3:	10: 0

Exa. 2.

(20)(12)(4)

ſh.	*d. qrs.*
13:	4: 3
7:	9: 3
4:	10: 2
18:	5: 3
0:	7: 2
1:	2: 1
7:	8: 3
16:	10: 1
Sum 3:	10: 11: 2

As I have ſet down, on the Top of each Denomination, the Numbers the Learner muſt carry at, I ſhall not further illuſtrate the above Examples; but, in the Column of Shillings, let him rather add the Units Place (without pointing) ſetting down the Exceſs over the 10's, and carrying one for each 10 to the Tens Place, and thence 1 for every 2 to the Place of *L.* And thus I have added the 2d Example.

The Farth. and Halfpence are frequently marked thus, $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$, which ſignify a Fourth of a Penny or 1 Farth. one Half of a Penny or 2 Farth. 3 Fourths of a Penny or 3 Farth. as in the following Example.

L.	*s.*	*d.*
120:	14:	$6\frac{1}{4}$
74:	8:	$7\frac{3}{4}$
12:	17:	$4\frac{1}{2}$
6:	4:	$8\frac{1}{4}$
17:	2:	$0\frac{1}{4}$
38:	15:	$5\frac{3}{4}$
319:	16:	$4\frac{1}{2}$
Sum 589:	19:	$1\frac{1}{4}$

Here for every $\frac{1}{2}$ *d.* I reckon 2 Farth. and add as in *Exa.* 1.

2. *Of* Troy *Weight.*

The Original of all Weights uſed in *England* (we are told) was a Grain of Wheat taken out of the middle of the Ear, and being well dried, 32 of them were to make 1 Penny-weight, 20 Penny-weight 1 Ounce, and 12 Ounces 1 Pound; but the Penny-weight was afterwards divided into 24 equal Parts called Grains, which is the leaſt Weight now commonly uſed. By this Weight are weighed Jewels, Gold, Silver, Liquors and Bread. Formerly Bz

TABLE.

Gr.			
24	1 d. wt.		
480	20	1 Oz.	
5760	240	12	1 lib.

That is, 24 Grains make 1 d. wt. 20 d. wt. or 480 Gr. make 1 Oz. 12 Oz. or 240 d. wt. or 5760 Gr. make 1 lib. Therefore you muſt carry or point at every 24, 20, 12, as in the following Example.

	(12)	(20)	(24)
	lib.	oz.	d.wt.	gr.
	148:	11:	19:	23
	25:	7:	6:	18
	10:	10:	14:	6
	44:	3:	12:	8
	31:	11:	8:	15
	18:	9:	10:	17
Sum	280:	6:	12:	15

Bz a peck loaf weighs 17 lb 6 oz 1 dr 2 w
a half peck 8 lb 11 oz 0 dr 3 gr
a quartern 4 lb 5 oz 8 dr
a ſack of Flour 280 lb

3. *Of Apothecaries Weight.*

This is the ſame with *Troy* Weight, unleſs that they divide the Lib. after a different manner, *viz.* into Grains, Scruples, Drams and Ounces. By this Weight the Apothecaries compound their Medicines, but buy and ſell their Drugs by *Aver-du-poiſe* Weight.

TABLE *for Apothecaries Weight.*

Gr.				
20	1 Scr.			
60	3	1 Dr.		
480	24	8	1 Oz.	
5760	288	96	12	1 Lib.

That is, 20 Gr. make 1 Scruple, 3 Scr. 1 Dr. 8 Dr. 1 Oz. and 12 Oz. 1 Lib. I uſe theſe Contractions, Gr. Scr. Dr. Oz. Lib.

Example.

(12)	(8)	(3)	(20)	
Lib.	oz.	dr.	ſcr.	gr.
36 :	11 :	7 :	2 :	19
0 :	8 :	2 :	0 :	14
7 :	10 :	3 :	2 :	6
2 :	7 :	0 :	1 :	12
3 :	6 :	5 :	1 :	16
1 :	10 :	6 :	2 :	15
Sum 53 :	7 :	3 :	0 :	2

one lb averdupois is equal to 14. 11. 16 troy

4. *Of* Aver-du-poiſe *Weight.*

By this Weight are weighed all coarſe Commodities, and ſuch as are ſubject to Waſte, as Iron, Lead, Salt, Hemp, Flax, Butter, Cheeſe, *&c.* It is divided into two Denominations, *viz.* the Greater and the Leſſer: The Greater conſiſts of Tuns, Hundred Weights, Quar-

Quarters and Pounds, which are thus mark'd, T. C. qrs. lib. The Leſſer conſiſts of Stones, Pounds, Ounces and Drams, thus marked, St. lib. oz. dr.

TABLE *for the Greater.*

lib.			
28	1 qr.		
112	4	1 C.	
2240	80	20	1 Tun.

That is, 28 lib. make 1 qr. 4 qrs. 1 C. and 20 C. 1 Tun.

TABLE *for the Leſſer.*

dr.			
16	1 oz.		
256	16	1 lib.	
4096	259	16	1 Stone.

That is, 16 dr. make 1 oz. 16 oz. 1 lib. and 16 lib. 1 St.

Examples.

	(20)	(4)	(28)
	T.	C. qrs.	lib.
	34:	19: 3:	27
	12:	10: 2:	18
	41:	5: 1:	8
	6:	11: 3:	14
	3:	9: 0:	20
	5:	13: 1:	16
Sum	104:	10: 1:	19

	(16)	(16)	(16)
	St.	lib. oz.	dr.
	70:	15: 15:	15
	18:	6: 12:	8
	45:	13: 6:	4
	20:	8: 3:	5
	6:	15: 8:	10
	17:	9: 11:	3
Sum	180:	5: 9:	13

Wool Weight in *England* hath theſe Denominations, *viz.* Lib. Cloves, Stones, Tod, Wey, Sack, Laſt, according to the following Table.

Lib.

Lib.						
7	1 Clove.					
14	2	1 Stone.				
28	4	2	1 Tod.			
182	26	13	6½	1 Wey.		
364	52	26	13	2	1 Sack.	
4368	624	312	156	24	12	1 Laft.

Mr. *Ward* ſays, that, by a very nice Experiment, he found, that 1 lib. *Aver-du-poiſe* is equal to 14 oz. 11 d. wt. 15½ gr. *Troy* Weight.

5. *Of Liquid Meaſures.*

Liquid Meaſure is founded on *Troy* Weight; for 8 lib. *Troy* Weight, taken out of the middle of the Ear and well dried, was to make a Gallon of Wine, Ale or Corn, than which no other Meaſure was at firſt allowed, tho' others have been introduc'd afterwards.

TABLE *of Wine Meaſure.*

Pints.						
2	1 Quart.					
8	4	1 Gall.				
336	168	42	1 Tierce.			
504	252	63	1½	1 Hhd.		
672	336	84	2	1⅓	1 Puncheon.	
1008	504	126	3	2	1½	1 But or Pipe.
2016	1008	252	6	4	3	2	1 Tun.

C

Accounts

Accounts are kept in these Denominations, Tuns, Pipes. hhds. gall. pints. Thus,

T.	P.	hhds.	gal.	pts.
54:	1:	1:	62:	7
3:	0:	1:	15:	4
1:	1:	0:	12:	7
2:	0:	0:	18:	4
3:	1:	1:	25:	3
2:	0:	1:	19:	6

10 Gall make 1 anker brandy

Note, 18 Gall. make 1 Rundlet, and 31 ½ Gall. make a Wine or Vinegar Barrel.

In the Excise the Wine Gall. is supposed to contain 231 Cubic Inches, consequently the Pint must contain 28 7/8 Cubic Inches; but the Beer or Ale Gallon is supposed to contain 282 Cubic Inches, and therefore the Pint must contain 35 ¼ such Inches.

TABLE *of Ale and Beer Measure.*

Pints.						
2	1 Quart.					
8	4	1 Gall.				
64	32	8	1 Firkin.			
128	64	16	2	1 Kilderkin.		
256	128	32	4	2	1 Barrel.	
384	192	48	6	3	1 ½	1 Hhd.
					3	2	1 Butt

A Firkin of Soap or Herrings is the same with that of Ale. 9 Gallons make a Firkin of Beer.

9 Gall are now reckoned firkin so that beer masure is ⅛ larger than it used to be

Exa.

Exa. in Ale Measure.

	(48) (4) (2)	
	hhd. gall. qt. pt.	
	3 : 47 : 3 : 1	
	1 : 18 : 2 : 0	
	0 : 28 : 1 : 0	
	0 : 13 : 3 : 1	
	0 : 26 : 2 : 1	
	2 : 15 : 1 : 1	
Sum	9 : 6 : 2 : 0	

In Beer Measure.

	(54) (4) (2)	
	hhd. gall. qt. pt.	
	13 : 53 : 3 : 1	
	8 : 42 : 1 : 0	
	3 : 16 : 2 : 1	
	1 : 24 : 2 : 1	
	2 : 15 : 3 : 1	
	1 : 48 : 2 : 0	
Sum	31 : 39 : 3 : 0	

In *Scotland* Accounts of Liquid Measure are kept in these Denominations, hhds. gall. pints, mutchkins, gills, according to the following Table.

Gills.

4	1 Mutch.			
16	4	1 Pint.		
128	32	8	1 Gall.	
2048	512	128	16	1 Hhd.

Note, 2 Mutchkins make 1 Chopin, and 2 Pints 1 Quart. The *English* Pint is somewhat larger than the *Scotch* Mutchkin.

Example.

	(16) (8) (4) (4)	
	hhd. gall. pt. mut gills.	
	1 : 15 : 7 : 3 : 3	
	0 : 8 : 4 : 1 : 0	
	3 : 12 : 4 : 2 : 2	
	2 : 4 : 6 : 3 : 1	
	1 : 0 : 5 : 2 : 2	
	4 : 10 : 3 : 1 : 3	
Sum	14 : 5 : 0 : 2 : 3	

6. *Of* Engliſh *Dry (or Corn) Meaſure.*

Dry Meaſure is different both from Wine and Ale Meaſure; for the Gall. here contains only 268 $\frac{4}{5}$ Cubic Inches, and the Buſhel 2150.42. The Denominations in this Meaſure are according to the following Table.

Note, 5 Pecks make a Buſh. Water Meaſure.

Ward ſays, that 10 Quarters make a Wey, and 12 Weys a Laſt, but all other Authors have it as in the annex'd Table.

Pints.	1 Quart.	1 Pottle.	1 Gall.	1 Peck.	1 Buſh.	1 Coomb.	1 Quarter.	1 Chald.	1 Tun or W.	1 Laſt.
2	1									
4	2	1								
8	4	2	1							
16	8	4	2	1						
64	32	16	8	4	1					
256	128	64	32	16	4	1				
512	256	128	64	32	8	2	1			
2048	1024	512	256	128	32	8	4	1		
2560	1280	640	320	160	40	10	5	$1\frac{1}{4}$	1 Tun or W.	
5120	2560	1280	640	320	80	20	10	$2\frac{1}{2}$	2	1 Laſt.

Accounts of this Meaſure are kept in Chald. qrs. buſh. pecks, gall. pts.

Example.

	(4)	(8)	(4)	(2)	(8)
	Ch.	qrs.	buſh.	p.	gal. pt.
	13:	3:	7:	3:	1: 7
	2:	0:	4:	2:	0: 4
	1:	2:	6:	2:	1: 0
	0:	2.	5:	1:	0: 3
	5:	3:	1:	2:	1: 6
	0:	1:	2:	1:	0: 4
Sum	24:	2:	4:	2:	0: 0

In

In *Scotland* the Denominations of Dry Measure are, Chalders, Bolls, Firlots, Pecks, Lipies, and in these Accounts are kept.

The *Engliſh* Buſhel is leſs than the *Scotch* Firlot, for 5 Buſhels make 1 Boll *Scotch* Meaſure.

Lipies.				
4	1 Peck.			
16	4	1 Firlot.		
64	16	4	1 Boll.	
1024	256	64	16	1 Chald.

Example.

	(16)	(4)	(4)	(4)
	Ch.	bolls.	fir.	peck.	lip.
	2 :	15 :	3 :	3 :	3
	1 :	10 :	2 :	0 :	2
	4 :	4 :	1 :	1 :	3
	2 :	12 :	2 :	2 :	1
	3 :	5 :	2 :	3 :	3
	11 :	5 :	1 :	2 :	0
Sum	26 :	6 :	2 :	2 :	0

7. *Of* Engliſh *Meaſures of Length.*

The Original of Meaſures of Lengths (as of Weights) is from a Corn of Wheat taken out of the middle of the Ear and well dried, 3 of theſe in Length were to make 1 Inch, and thence all other Meaſures are computed, as in the following Table.

Bar. Corns.	1 Inch.	1 Foot.	1 Yard.	1 Pole or Perch.	1 Furlong.	1 Mile.
3	1 Inch.					
36	12	1 Foot.				
108	36	3	1 Yard.			
594	198	16½	5½	1 Pole or Perch.		
23760	7920	660	220	40	1 Furlong.	
190080	63360	5280	1760	320	8	1 Mile.

6 feet 1 Fathom
3 miles 1 league
60 miles 2 degrees

Accounts are kept in Miles, furlongs, yards, feet and inches.

Exa.

	(8)	(220)	(3)	(12)
M.	fur.	yds.	feet	in.
1 :	7 :	219 :	2 :	11
0 :	4 :	140 :	1 :	8
0 :	2 :	17 :	0 :	5
3 :	3 :	200 :	2 :	1
2 :	4 :	175 :	1 :	8
1 :	6 :	134 :	2 :	7
Sum 10 :	6 :	8 :	2 :	4

Cloth Measure consists of these Denominations,, yds, qrs. nails.

Exa.

	(4)	(4)
yds.	qrs	nai.
43 :	3 :	3
6 :	2 :	1
7 :	0 :	2
4 :	1 :	3
5 :	2 :	1
13 :	1 :	2
Sum 81 :	0 :	0

The easiest method of adding the above yds. is to begin with the Units Place, carrying at 10's, and then adding the other 2 Columns together, carrying at 22.

Note, That 1¼ yd. or 5 quarters, or 45 inches, make an Ell *English*, also 2 yds make a Fathom.

The

The *Scotch* Ell contains 37 *Scotch* Inches, and is divided into 4 Quarters, and a Quarter into 4 Nails. The Foot Measure contains 12 of those Inches, whereof the Ell contains 37. The *Scotch* Ell is $\frac{1}{3}$ Part of an Inch *English* more than 37 Inches *English*; so that the *Scotch* Foot is to the *English* as 186 to 185.

The *Scotch* Mile contains 8 Furlongs, a Furlong 40 Falls, and a Fall 18 $\frac{1}{2}$ Feet *Scotch*, according to the following Table.

Inches.					
12	1 Foot.				
37	$3\frac{1}{12}$	1 Ell.			
222	$18\frac{1}{2}$	6	1 Fall.		
8880	740	240	40	1 Furlong.	
71040	5920	1920	320	8	1 Mile.

Thus the *Scotch* Mile is 5920 *Scotch* Feet; so that if the Difference betwixt the *English* and *Scotch* Feet is not regarded, the *Scotch* Mile will be to the *English* as 37 or 33, that is 37 *English* Miles are equal to 33 *Scotch Miles*.

Of Land Measure.

The Denominations used in measuring of Land, are, Poles, Roods, Acres, Miles, and are always understood to be square.

40 square Poles make 1 Rood, 4 Roods an Acre, and 640 Acres a Mile. See the following Table.

Sq.

sq. Inches.				
144	1 sq. Foot.			
1296	9	1 sq. Yd.		
39204	272¼	30¼	1 sq. Pole or Perch.	
1568160	10890	1210	40	1 sq. Furlong or Rood.
6272640	43560	4840	160	4 1 sq. ~~Mile.~~

TABLE *for* Scotch *Superficial Measures.*

square Inches.				
144	1 sq. Foot.			
1260	9 73/144	1 sq Ell.		
49284	342¼	26	1 sq Fall.	
1971360	13690	1410	40	1 Rood.
7885440	54760	5760	160	1 Acre.

A *Scotch* Acre is to an *Engliſh* Acre as 55353.6 to 43560.

8. *Of Time.*

The moſt common Diviſion of Time is into Years, Months, Days, Hours, Minutes, Seconds, Thirds, *&c.* but Seconds being the leaſt Part of Time which can be meaſured, with them I begin the following Table.

Seconds.				
60	1 Minute.			
3000	60	1 Hour.		
86400	1440	24	1 Day.	
31556937	525949	8765	365 D. 5 Ho. 48 Min. 57 Sec.	1 Year.

But

But for eafe in Calculation, we make the Year to conſiſt of 365 Days, 6 Ho. and in Leap (or every 4th) Year, of 366 Days. Therefore 60 Min. make 1 Ho. 24 Ho. 1 Day, 28 Days 1 Mon. and 13 Months and 1 Day make one Year; this is called the Common Year, and is divided into 12 unequal Months, *viz.*

January	31 Days	*July*	31
February	28 and in Leap Year 29	*Auguſt*	31
March.	31	*September.*	30
April	30	*October.*	31
May	31	*November.*	30
June	30	*December.*	31

Some Things are accounted by the Dozen, the higheſt Denomination whereof is a Groſs. Thus, 12 Particulars make 1 doz. and 12 doz. 1 Groſs.

Example.

	Groſs.	doz.	part.	
	4:	11:	11	
	1:	8:	7	
	0:	2:	4	
	2:	5:	5	*Note*, 12 Barrels make 1 Laſt.
	1:	10:	8	
	3:	6:	3	
Sum	14:	9:	2	

of

Of the Proof of Addition.

1. In order to prove the Work of Addition, you may distribute the Numbers added into several Parcels; and after having added these Parcels together, if their Sum is the same with that first found, the Work has been right. To illustrate this I shall repeat *Exa.* 5.

26370	
285	
	26655
186	
400	
76	
	662
42	
27	
112	
	181
Total	27498

2. Addition may be proved by casting out the 9's, thus: Take each of the given Numbers separately, and add their Figures together as simple Units, and when the Sum is equal to or greater than 9, but less than 18, neglect the 9, adding what is over to the next Figure, and so proceed 'till you have gone thro' them all, marking what is over or under 9 at the last Figure: Do the same with all the Numbers given to be added, setting down the Excesses of 9 together in one Line, and sum them up the same way, marking the Excess of 9 as before, or what the Sum is less than 9. Lastly, do the same with the Total Sum, and what is under 9, or over any Number of 9's in this, must be equal to the Excess or Number less than 9 last marked, else the Work is false.

Exa.

	Excess of 9's
4768	7
1378	1
793	1
574	7
7513	7

Here I say 4+7=11, which is 2 over, then 2+6+8=16, which is 7 over, and therefore I set it down over against this first Line; and thus proceeding, I find the Excesses over 9's in the other Lines to be 1, 1, 7; the Sum of all which is 16, from which casting away 9, there remains 7, which I set down under; and casting out the 9's the same way from the total Sum 7513, I find 7 over also;

alſo; wherefore the Work is right. But this being tedious, eſpecially where there are many Particulars,

3. The beſt Method, in my Opinion, is to begin at the Top and add downwards, whereas it was formerly done by beginning below.

Queſtions to exerciſe the Learner in this Rule.

Qu. 1. I bought a Parcel of Goods, the firſt Coſt whereof was *L.* 103 : 15. I paid for Packing 8 *ſh.* 6 *d.* for Carriage *L.* 1 : 2 : 4. and at Bargaining 4 *ſh.* 8 *d.* What did the Goods ſtand me in, in all?

L.	*ſh.*	*d.*	
103 :	15 :	0	firſt Coſt.
0 :	8 :	6	Packing.
1 :	2 :	4	Carriage.
0 :	4 :	8	Bargaining.
105 :	10 :	6	Tot. Charge.

Qu. 2. I lent my Friend at one Time *L.* 48 : 10. at another *L.* 100. at another 18 *ſh.* 10 *d.* at another *L.* 31 : 14. at another *L.* 40 : 8 *d.* How much comes it to in all?

L.	48 :	10 :	
	100 :		
		18 :	10
	31 :	14 :	
	40 :	:	8
L.	221 :	3 :	6 *Anſwer.*

Qu. 3. A Father was 22 Years 6 Mon. old (reckoning 28 Days to a Mon. and 13 Mon. to a Year) when his eldeſt Son was born; betwixt the eldeſt and ſeconp were 11 Mon. 12 Days; betwixt the ſecond and

and third were 2 Years 8 Mon. when the third is 14 Years 2 Mon. how old is the Father?

Y.	M.	D.
22:	6:	00
0:	11:	12
2:	8:	00
14:	2:	00
40:	1:	12 *Answer.*

Qu. 4. *A* has such a Sum of Money, that if *L.* 13: 15: 8 be taken from it, what remains is equal to *L.* 61; 10: 7.

L.		
13:	15:	8
61:	10:	7
75:	6:	3 *Answer.*

CHAP. III. *Of* SUBTRACTION.

BY this Rule we find the Difference betwixt any two Numbers or Sums proposed.

The Number from which the Subtraction is to be made is called the *Minuend*, the Number to be subtracted is called the *Subtrahend*, and what remains is called the *Remainder*, *Difference* or *Excess*.

Exa

Exa.

Minuend 6538	From 3867
Subtrahend 4126	Take 1645
Rem. 2412	Difference 2222

When the Figure to be ſubtracted happens to be greater than that which ſtands over it, add 10 to the upper Figure, and then ſubtract or take the lower Figure from the Sum of both, paying 1 to the next low Figure for the 10 thus borrowed.

Exa.

What is the Difference betwixt 4632 and 3654?

4632
3654
978 *Anſr.*

From 179864	420	3000	2713
Take 57698	180	2816	1080
Rem. 122166	240	184	1633

I proceed next to give Examples in Mixt Applicate Numbers, where you'll mind to borrow (in caſe you want) as you carried in Addition.

Exa. in Money.

	L.	*s.*	*d.*	*f.*
From *L.* 147:	13:	6:	3	
Take 89:	10:	4:	2	
Rem. 58:	3:	2:	1	

	L.	*s.*	*d.*	*f.*
From 1245:	8:	4:	1	
Take 486:	17:	8:	3	
Rem. 758:	10:	7:	2	

Troy *Weight.*

	lib.	oz.	d.	wt.	gr.
From	142:	7:	2:	15	
Take	128:	4:	14:	20	
Rem.	14:	2:	7:	19	

Aver-du-p. *Weight Leſſer.*

	ſt.	lib.	oz.	dr.
From	18:	12:	12:	10
Take	6:	10:	13:	14
Rem.	12:	1:	14:	12

Apothecaries Weight.

	lib.	oz.	dr.	ſcr.	gr.
From	36:	4:	5:	1:	12
Take	15:	8:	6:	0:	14
Rem.	20:	7:	7:	0:	18

Engliſh *Liquid Meaſure.*

	T.	P.	hhd.	gal.	pt.
From	20:	1:	0:	20:	6
Take	15:	0:	1:	40:	4
Rem.	5:	0:	0:	43:	2

Aver-du-poiſe *Weight Gr.*

	T.	C.	qrs.	lib.
From	40:	15:	2:	20
Take	18:	16:	1:	25
Rem.	21:	19:	0:	23

Scots *Liquid Meaſure.*

	hhd.	gal.	pt.	mut.	gil.
From	150:	12:	2:	1:	0
Take	86:	10:	6:	3:	2
Rem.	64:	1:	3:	1:	2

Engliſh *Dry Meaſure.*

	Ch.	qrs.	buſh.	p.	gal.	pt.
From	4:	2:	6:	2:	0:	4
Take	1:	3:	4:	3:	1:	6
Rem.	2:	3:	1:	2:	0:	6

Scots *Dry Meaſure.*

	Ch.	B.	fir.	p.	lip.
From	5:	8:	2:	1:	2
Take	4:	12:	3:	2:	3
Rem.	0:	11:	2:	2:	3

Long

Long Measure.	M. fur. yds. feet in.	*Cloth Measure.*	yds. qrs. n.
From	12: 4: 124: 1: 10	From	45: 2: 1
Take	8: 7: 215: 1: 11	Take	18: 3: 3
Rem.	3: 4: 128: 2: 11	Rem.	26: 2: 2

Queſtions to exerciſe the Learner in this Rule.

Qu. 1. A Man was born in the Year of our LORD 1686, how old is he this preſent Year 1738?

1738
1686
Anſr. 52

Qu. 2. Suppoſe the ſame Perſon ſhould live to the Year 1760, how old will he be then?

1760
1686
74 *Anſ.*

Qu. 3. I lent my Friend *L.* 375: 14: 6. whereof he has paid me *L.* 198: 16: 8. how much does he yet owe me?

L. 375: 14: 6 Lent.
198: 16: 8 Received.
176: 17: 10 *Anſr.*

Qu. 4. I bought 50 Tuns 5 C. 1 qr. 20 lib. 10 oz. of Iron, whereof I have ſold 24 Tuns 8 C. 17 lib. 12 oz. How much remains unſold?

	T.	C.	qr.	lib.	oz.
Bought	50:	5:	1:	20:	10
Sold	24:	8:	0:	17:	12
Unſold	25:	17:	1:	2:	14

Qu. 5. A Church is 215 Years ſtanding this preſent Year, what Year was it built in?

1738
215

1523 *Anſwer.*

Qu. 6. What Sum of Money added to *L.* 130: 17 will make it *L.* 413: 12: 7?

413: 12: 7
130: 17:

282: 15: 7 *Anſwer.*

This Queſtion is the ſame, as if a Man was owing *L.* 413: 12: 7 to be paid preſently, and had only *L.* 130: 17, and it ſhould be demanded, how much he muſt borrow preciſely to pay the ſame.

Qu. 7. Lent my Friend at one Time *L.* 172: 13: 4, at another Time *L.* 20: 12, at another Time *L.* 56: 16, whereof he has paid me *L.* 209. What does he yet owe me?

L. 172: 13: 4
20: 12: 0
56: 16: 0

250: 1: 4 Lent in all.
209: Paid me.

Anſr. 41: 1: 4 Sum yet due to me.

Qu.

Qu. 8. There are two Numbers, the greater of which is 128, and their Difference is 57 What is the lesser?

128
57
71 *Answer.*

Qu. 9. The Sum of the Ages of two Men is 140 ½ Years, and the Difference is 47 Years 9 Mon. What is the Age of each? (accounting 12 Mon. to the Year.)

	Y.	M.			
	140:	6	70:	3	} Add
Half the Sum is	70:	3	23:	10½	
	47:	9	94:	1½	Elder's Age.
Half the Difference	23:	10½			
			70:	3	} Sub.
94:	1½		23:	10½	
46:	4½				
140:	6	Proof.	46:	4½	Younger's Age.

Qu. 10. A Father was 21 Years 10 Months and 6 Days old when his eldest Son was born, and is now 68 Years 1 Month 20 Days; how old is the Son? (accounting 30 Days to the Mon. and 12 Mon. to the Year.)

Years	m	Days
68:	1:	20
Father 21:	10:	6
46:	3:	14 *Answer.*

Proof of the Work of Subtraction.

This may be made two Ways, *viz.* 1. By adding the Subtrahend to the Remainder, for the Sum ought to be the ſame with the Minuend. Or, 2. By ſubtracting the Remainder from the Minuend, for this Difference ought to be the ſame with the Subtrahend.

CHAP. IV. *Of* MULTIPLICATION.

MUltiplication is the taking or repeating any Number a certain number of Times.

TABLE.

1	2	3	4	5	6	7	8	9	12
2	4	6	8	10	12	14	16	18	24
3	6	9	12	15	18	21	24	27	36
4	8	12	16	20	24	28	32	36	48
5	10	15	20	25	30	35	40	45	60
6	12	18	24	30	36	42	48	54	72
7	14	21	28	35	42	49	56	63	84
8	16	24	32	40	48	56	64	72	96
9	18	27	36	45	54	63	72	81	108
12	24	36	48	60	72	84	96	108	144

This

This Table the Learner ought to get by Heart perfectly before he proceed.

The Number to be multiplied is called the *Multiplicand*, that Number which multiplies is called the *Multiplier*, and the Number reſulting from the multiplying the one by the other is called the *Product*. The firſt two are otherwiſe called *Factors*.

I. *To Multiply Abſtract Numbers.*

RULE. Firſt ſet down the Multiplicand, and under it orderly the Multiplier, and beginning with the Figure which ſtands in the Units Place of the Multiplier, by it multiply the Figure ſtanding above it in the Multiplicand; and if the Product is leſs than 10, ſet it down in its proper Place (and proceed to multiply the next Figure of the Multiplicand;) but if the Product is above 10 or 10's, ſet down only the Exceſs, and carry the 10 or 10's to the Product of the next Figure when multiplied, ſetting down the Exceſs as before: And thus go on to multiply all the Figures of the Multiplicand by this firſt Figure of the Multiplier; then multiply the Multiplicand by the ſecond Figure of the Multiplier (if this laſt conſiſts of more Places than one) after the ſame manner, ſetting down the Product as before, and proceed to multiply by each Figure of the Multiplier till you have done; only mind to ſet down the firſt Figure of the Product of each particular Multiplication under the multiplying Figure; then adding all the partial Products together, the Sum is the Anſwer. The Rule will be better underſtood by attending to the Operation of the following Examples.

Exa. 1.

Multiplicand 4687
Multiplier 4

Product 18748

Exa. 2.

17468
7

122276 Product.

Exa. 3.

Multiply 3576
by 12

Product 42912

Exa. 4

18694
24

74776 Product by 4.
37388 Product by 2.

448656 Total Product.

Exa. 5.

7365
35

36825 Product by 5.
22095 Product by 3.

257775 Total Product.

Exa. 6.

8065
68

64520
48390

548420 Product.

Exa. 7.

25968
769

233712
155808
181776

19969392

Exa. 8.

39745
4673

119235
278215
238470
158980

185728385

1. When

1. When the Multiplier or Multiplicand, or both, end in a o or o's, write the ſignificant Figures under one another, and multiply, neglecting the o's 'till you have done; but then you muſt add as many o's to the Product when found as are contained in both Factors.

	Exa. 1.	*Exa.* 2.	*Exa.* 3.
Mult.	3260	325	38000
by	24	240	2400
	1304	1300	152
	652	650	76
	78240	78000	91200000

2. When a o or o's happen to ſtand between ſignificant Figures in the Multiplier, neglect them altogether.

Exa. 1.	*Exa.* 2.
4376	13706
608	6008
35008	109648
26256	82236
2660608	82345648

3. When the Multiplier is 1 with any Number of o's annexed, *viz.* 10, 100, 1000, *&c.* you have only to add to the Multiplicand as many o's as are contained in the Multiplier, and the Work is done

Thus, 486×10 = 4860
486×100 = 48600
486×1000 = 486000 *&c.*

4. It

4. It is convenient to make the leaſt of the two given Numbers Multiplier, tho' it may be propoſed as Multiplicand, ſeeing the Product will be the ſame.

Exa. Let it be propoſed to multiply 375 by 4836.

Operation.

4836	
375	
24180	
33852	
14508	
1813500	

The Multiplier may ſtand uppermoſt, thus,

45	
4837	
29022	
19348	
222502	

5. When the Multiplier is a Number that has 9 in all its Places, as 9, 99, 999, *&c.* ſet as many 0's on the right of the Multiplicand as there are 9's in the Multiplier, and ſubtract it from itſelf ſo increaſed. Thus if 346 was to be multiplied by 9, 99, 999 or 9999, the Products would be (as by the following Operations)

3460	34600	346000	3460000
346	346	346	346
3114	34254	345654	3459654

6. When either Factor conſiſts of 2 Figures, which are the Product of any 2 Digits, as 24=4 times 6, multiply by one of theſe Digits, and the Product by the other, and the laſt Product is the Anſwer: Thus to multiply 8475 by 24, the Operation will ſtand as below.

8475

8475
4
33900
6
203400 Product.

Exa. 2. Multiply 346 by 54, which is 6 times 9.

6
2076
9
18684 Product.

And we may use the same Method when the Multiplier is the Product of any 3 or more Digets: For Exa. to multiply 346 by 168, which is 4 times 6 times 7. Thus found,

4 | 168
42
6 | 5
4

346
4
1384
6
8304
7
58128 Product.

You'll see the Use of such Multiplication further on.

II. *To*

II. *To Multiply Mixt Applicate Numbers.*

When the Multiplicand is Mixt and Applicate, and the Multiplier a ſingle Abſtract Number, or the Product of two or more Figures, the Operation may be performed without reducing the Multiplicand to the loweſt Species mentioned, as you'll ſee by the following Examples.

Multiply	*L.* 14:	12:	8:	3	Here I carry 1 for every
by				7	4, for every 12, for every
	102:	9:	1:	1	20, in the Place of *far. d.* and *ſh.* reſpectively.

Exa. 2. Multiply

St.	lib.	oz.	dr.	
10:	4:	13:	6	by 8.
			8	
82:	6:	11:	0	Here I carry 1 for every 16, 16, 16. *Vide* Table 2. *p.* 12.

Exa. 3. Where the Multiplier is the Product of two Digits;

Multiply	*L.* 17:	10:	7:	2	45 is 5 times 9.
by 45				5	
	87:	13:	1:	2	
				9	
	788:	18:	1:	2	*Anſwer.*

Exa.

St. lib. oz. dr.

Exa. 4. Multiply 25: 11: 6: 10 28 is 4 times 7.
by 28. 4

102: 13: 10: 8
7

719: 15: 9: 8 *Answer.*

Ch. b. fir. p.lip.

Exa. 5. Multiply 5: 12: 2: 1: 3
by 224. 4 224 is 4×7×8

4 | 224
56
8
1

23: 2: 1: 3: 0
7

162: 1: 0: 1: 0
8

1296: 8: 2: 0: 0 *Answer.*

By duly attending to the Method of operating these 5 Examples, you may easily perform others of the same Nature. Wherefore I proceed to

REDUCTION

From an higher to a lower Denomination (called *Reduction Descending*) performed by Multiplication.

Money.

In *L.* 374: 14: 10: 1 how many Farthings?

20
7494 *sh.*
12
89938 *d.*
4
359753 *far.*

Here I multiply the *L.* by 20, and the Product is *sh.* this I multiply by 12, and the Product is *d.* which last I multiply by 4, and the Product is *far.* But note, that I take in the odd *sh. d.* and *far.* in their proper Places as I go along.

Troy *Weight.*

lb oz dwt gr

In 148 : 4 : 10 : 15 how many Grains?

148 : 4 : 10 : 15

12

1780 oz.

20

35610 d. wt.

24

142445
71221

854655 gr. *Anſwer.*

Here I multiply the lib. by 12 to bring them to oz. theſe by 20 to reduce them to d. wt. and theſe laſt by 24 for gr.

Aver-du-poiſe Weight Greater.

T. C. qrs. lib.

In 120 : 12 : 2 : 17 how many lib.

Anſr. 270217 lib.

Aver-du-poiſe Weight Leſſer.

St. lib. oz. dr.

In 40 : 11 : 12 : 7 how many Drams?

Anſr. 166855.

Engliſh *Liquid Meaſure.*

T. P. hhd. gal. pts.

In 17 : 0 : 1 : 44 : 5 how many pints?

Anſr. 35133

Scots *Liquid Meaſure.*

hhd. gal. pt. mut. gil.

In 2 : 13 : 5 : 2 : 1 how many Gills?

Anſr. 5849.

Engliſh

English *Dry Measure.*

Ch. qrs. bush. p. gal. pt.
In 15 : 2 : 7 : 2 : 1 : 4 how many Pints?
Answ. 32236.

Scots *Dry Measure.*

Ch. bol. fir. p. lip.
In 18 : 15 : 2 : 3 : 3 how many Lipies?
Answ. 19439.

I have left the Operation of the preceding Questions to the Learner's Exercise.

Questions in Multiplication.

Qu. 1. If any one Thing cost 4 *sh.* what is the Value of 42 such Things at the same Rate?

42
4
---
168 *sh.* *Answ.* or *L.* 8 : 8

Qu. 2. If I spend 7 *sh.* a Day, how much is it a Year?

365
7

L. sh.
2555 *sh.* *Answ.* 127 : 15

Qu. 3. If 63 gall. make a hhd. how many gall. are there in 40 hhds.

40
63

2520 *Answ.*

Qu. 4. If 1 C. of any thing coſt *L.* 1: 16: 4 what is the value of 8 C. at the ſame Rate?

L. 1: 16: 4
8

14: 10: 8 *Anſr.*

Qu. 5. If any one Thing coſt *L.* 3: 15: 10 how much will 28 ſuch Things coſt at the ſame Rate?

L. 3: 15: 10
4

Here I multiply by 4 and 7, inſtead of 28.

15: 3: 4
7

L. 106: 3: 4 *Anſr.*

Qu. 6. A Father gave his Daughter for her Dowry 24 Boxes, and in each Box were 16 leſſer Boxes, in each leſſer Box were 12 Purſes, and in each Purſe were 16 leſſer Purſes, in each of which laſt were 4 *d. Scots.* What was her Portion?

24
16

384 Number of leſſer Boxes.
12

4608 Number of bigger Purſes.
16

27648
4608

73728 Number of leſſer Purſes.
4

294912 *d. Anſr.* or *L.* 1228: 16 found by dividing by 12 and 20.

Qu.

Qu. 7. I have 20 Casks of Raisins, the net Weight of each is 7 Stone, 9 lib. 10 oz. what is the net Weight of the whole?

St. 7: 9: 10
4
30: 6: 8 Here I multiply by 4 and 5 instead of 20.
5
152: 0: 8 *Answ.*

Qu. 8. How many *ß. d.* and Halfpence are contained in *L.* 10000 *Sterl.?*

10000 *L.*
20
200000 *ß.*
12
2400000 *d.*
2
4800000 Halfpence.

Qu 9. What Number divided by 8 gives 21 for the Quote?

21
8
168 *Answ.*

Qu. 10. What Sum of Money must be divided equally amongst 18 Men, so that each may receive, *L.* 14: 6: 8½.

L. 14 : 6 : 8½ 18 is 3×6.
3
43 : 0 : 1½
6
258 : 0 : 9 *Answer.*

Qu. 11. In 50 Gross 8 doz. of Pairs of Stockings, how many single Pairs?

Gr. doz.
50 : 8
12
608
12
7296 Pairs, *Answer.*

Qu. 12. What is the Total Weight of 26 Packs of Wool, each weighing 17 Stone 12 lib. 6 oz. net?

St. lib. oz.
17 : 12 : 6
4
71 : 1 : 8
6
426 : 9 : 0 for 24 times.
35 : 8 : 12 for 2 times.
462 : 1 : 12 *Answer.*

Qu. 13. What Number divided by 3, 5, 7, 12, will have no Remainder? *Answ.* 1260.

Proof of Multiplication.

Multiplication is best proved by Division; thus, divide the Product by the Multiplier, the Quote is the Mul-

Multiplicand; or, divide the Product by the Multiplicand, and the Quote is the Multiplier.

I ſhall finiſh this Rule, when I have obſerved, that both Factors cannot be Applicate Numbers, but that the one of them muſt neceſſarily be Abſtract. Wherefore it is abſurd to propoſe (for Exa.) *L.* 4: 6: 8 to be multiplied by *L.* 3: 2: 6. *&c.* If the Propoſers of ſuch Queſtions would be ſo good as to tell us, how oft they would have ſuch a Sum taken or repeated (which muſt be the meaning of Multiplication, elſe it has no meaning) I ſhould do my beſt to give them a ſatisfactory Anſwer; but 'till they explain themſelves, I think they deſerve none. My deſigned Brevity does not allow me to proſecute this Subject, and therefore I refer the Reader to *Malcolm*'s Arith. *pag.* 85. Edit. *Lond.* 1730.

CHAP. V. *Of* DIVISION.

DIviſion finds how oft one Number is contained in another, or what Part it is of the ſame.

The Number to be divided is called the *Dividend*, the Number which divides is called the *Diviſor*, and the Number reſulting by dividing the one by the other, is called the *Quotient* or *Quote*.

RULE.

Set down the Diviſor and Dividend as in the annexed Exa. and if the Diviſor is contained in an equal Number of Places of the Dividend, ſet a Point under or above that Figure of the Dividend; but if the Diviſor is not contained in an equal Number of Places of the Dividend, ſet the Point under or above that Figure which ſtands one Place further to the right; then conſider how often the Diviſor is contained in that Num-

Diviſor	Dividend	Quote
4)	7376	1844

Number of Places of the Dividend, and ſet down the Anſwer on the right (of) within the crooked Line, and multiplying the Diviſor by the ſaid Figure, place the Product orderly under the Dividual, and ſubtract it, writing the Remainder below: Then take down the next Figure of the Dividend, and annexing it to the right of the Remainder, conſider how oft the Diviſor is contained in that Sum, and ſet the Anſwer in the Quote, on the right of the Figure which ſtands there already, multiplying and ſubtracting as before; and to the Remainder take down the next Figure; and thus proceed till all the Figures are taken down, and the Quote completed.

Note 1. The Figure ſet in the Quote muſt expreſs the greateſt Number of Times the Diviſor is contained in that Part of the Dividend, and conſequently the Remainder muſt always be leſs than the Diviſor.

2. The Product of the Quote Figure multiplied into the Diviſor muſt never exceed the Dividual.

Exa. 1.

```
4) 7376 (1844
   4...
   ---
    33
    32
    --
     17
     16
     --
      16
      16
      --
```

Exa. 2.

```
6) 87493 (14582
   6....
   -----
    27
    24
    --
     34
     30
     --
      49
      48
      --
       13
       12
       --
      Rem. 1
```

Exa. 3.

8)201634 (25204
16····
—
41
40
—
16
16
——
34
32
—
Rem. 2

Exa. 4.

57) 38426 (674
342··
——
422
399
——
236
228
——
Rem. 8

Exa. 5.

457) 1246874 (2728
914···
——
3328
3199
——
1297
914
——
3834
3656
——
Rem. 178

Exa. 6.

1579) 1087647 (688
9474··
——
14024
12632
——
13927
12632
——
Rem. 1295

Exa.

Exa. 7.

63087)410907468(6513
378522
323854
315435
84196
63087
211098
189261
Rem. 21837

Exa. 8.

7469)170196846(22787
14938
20816
14938
58788
52283
65054
59752
53026
52283
Rem. 743

More Examples for Practice.

8174679) 417690850638 (51095 $\frac{5627133}{8174679}$
674100746) 192037642746 74 (28487 $\frac{630323372}{674100746}$
341968257644) 462873467474 67 (135 $\frac{12763196552 7}{341968257644}$
9610070067 3846)4173867474628468(43 $\frac{41537345653090}{9610070067 3846}$

Contractions in Division.

1. When the Divisor is a single Figure, or even 11 or 12, the Division may be perform'd by subtracting mentally, and the Quote found by one Line.

Exa. 1.

4) 7384
1846 Quote.

Exa. 2.

6) 327968
54661 $\frac{1}{3}$ Quote.

Exa.

Exa. 3.

8) 279685

34960$\frac{5}{8}$ Quote.

Exa. 4.

9) 1326754

147417$\frac{1}{9}$ Quote.

11) 428764

38978$\frac{6}{11}$ Quote.

12) 391467

32622$\frac{3}{4}$ Quote.

2. When the Diviſor is the Product of one or more ſingle Figures (or Digits) divide by any one of theſe, and the Quote by any other, and ſo on, and the laſt Quote is the Anſwer: Thus, if I was to divide 68357 by 24, I would work as in the Margin.

24 } 4 | 68357
     6 | 17089 5
       2848 $\frac{5}{24}$

Or thus,

24 } 6 | 68357
     4 | 11392 5
       2848 $\frac{5}{24}$

When any thing remains after the firſt Diviſion, it muſt be ſet down with a Line drawn through it, as you ſee in the annexed Exa. and if any thing remains over the next Diviſion, it muſt not be ſet down, but you are to multiply it (mentally) into the firſt Diviſor, adding the firſt Remainder to the Product for the true (total) Remainder; ſo 1 remaining after I divided by 6, I ſay once 4 is 4, which with the 1 remaining formerly makes 5 for the true Remainder, as you may prove by dividing the long Way.

Exa. 2. Let it be required to find the 63d Part of 39485.

63 } 7 | 39485
   9 | 5640 5
     626 $\frac{47}{63}$ Quote.

63 is =7×9

Exa. 3. Required the 27th Part of 16346

27 } 3 | 16346
   9 | 5448 2
     605 $\frac{11}{27}$ Quote.

27=3×9.

Exa. 4. Required the 64th Part of 16736.

64 } 8 | 16736
   8 | 2092 Ø
     261 $\frac{4}{8}$ or $\frac{1}{2}$ Quote.

Here the firſt Remainder being 0, I don't multiply the 2d Remainder 4 into the firſt Diviſor, becauſe there is nothing to add to it, and therefore the Remainder is the ſame as if it had been only a ſimple Diviſion by 8. However, the Remainder would have been equivalent, if I had multiplied it into the firſt Diviſor, according to the Rule, only it would have come come out in a higher Term, for $\frac{32}{64}$ is when reduced $\frac{4}{8}$ or $\frac{1}{2}$.

If you want to know the Reaſon of thus managing the Remainders, let us reſume the 3d Exa. where, after dividing by 3, there remains 2, which is $\frac{2}{3}$; then there is $5448\frac{2}{3}$ to be divided by 9, and after dividing the whole Part there remains 3, which reduced to a Fraction, having for its Denominator 3, (*viz.* the Denominator of the Fraction to be divided) makes $\frac{9}{3}$; to which adding the $\frac{2}{3}$ remaining over the firſt Diviſion, the Sum is $\frac{11}{3}$, this divided by 9 quotes $\frac{11}{27}$, as is plain from 3d *Caſe, Diviſ. Com. Fract.* But this laſt is not ſo eaſy as the firſt Method.

27 | 3 | 16346
   | 9 | 5448 $\frac{2}{3}$
   | | $605\frac{11}{27}$

Exa. 5.

75 | 3 | 63805
   | 5 | 21268 4
   | 5 | 4253 40
   | | 850 $\frac{55}{75}$

Here the firſt Remainder is 1, which I ſet down; the 2d Remainder is 3, which I multiply into the firſt Diviſor 3, and adding the firſt Remainder, the Sum is 10, which I mark; the 3d Remainder is alſo 3, which I multiply into the Product of the 2 firſt Diviſors, *viz.* 3 and 5, and it makes 45; to which adding the Remainder 10, the Sum is 55 for the total Remainder.

Exa. 6. Required to divide 3867395 by 756 after this contracted Method.

756

756	3	3867395
	6	1289131 2
	6	214855 5
	7	35809 23
		5115 $\frac{415}{756}$

Here in order to find the Diviſors, I divide 756 by 3, the Quote by 6, this by 6, and this by 7; and ſo of others: Thus,

3	756
6	252
6	42
7	7 .
	1

So I find that 3×6×6×7=756.

3. When the Diviſor ends in 0 or 0's, cut off theſe 0's and as many Placcs of the Dividend, and ſo the Operation will be ſhorter.

Exa. 1.

4|00) 864|00
216 Quote.

Exa. 2.

74|000) 4630|846 (62 Quote.
444
190
148
Rem. 42846

Exa. 3.

24	0	4	39401	6
	6	9850 4
		1641 $\frac{176}{240}$ Quote.

Exa. 4.

45	00	5	179860	00
	9	35972 ø
		3996 $\frac{8}{9}$ Quote.

4. When both Diviſor and Dividend can be reduced lower, reduce, and the Work will be eaſier: Thus, 468) 36840 (

4 468)	4 36840
3 117)	3 9210
39)	3070 (78
	273
	340
	312
Rem.	28

Here firſt I divide both by 4, and theſe Quotes by 3, and the Diviſor becomes 39, and the Dividend 3070.

Sometimes by this Method the Diviſor becomes 1; Thus, to divide 11232 by 432.

6 432)	6 11232	So the Quote is 26 for 1) 26 (26
9 72)	9 1872	times.
8 8)	8 208	
1)	26	

5. Diviſion may be contracted, by omitting to write down the ſeveral Products of the Quote Figures into the Diviſor, but ſubtracting mentally, and ſetting down only the Remainders, as in Exa. 4, and 5, repeated.

Exa. 4.

57) 38426 (674
422··
236
Rem. 8

Exa. 5.

457) 1246874 (2728
3328··
1297
3834
Rem. 178

Some daſh a Line through each Figure of the Dividual as they ſubtract; thus, in Exa. 4 repeated.

57) 38426 (6
42

57) 38426 (67
423

57) 38426 (674
4238
2

Exa. 5. repeated.

457) 1246874 (2728
3328
1297
3834
178

Note, the Figures 178 wanting the Daſh, are the Remainder.

Diviſion in Mixt Applicate Numbers.

Case I. When the Dividend is a Simple or Mixt Applicate Number, and the Diviſor Abſtract: See the following Examples.

L.		*L.*	*s.*	*d.*	*f.*
34) 2629 (77 *L.*		76) 2747 :	16 :	10 :	3 (36 *L.*
238		228			
249		467			
238		456			
11 Rem.		11			
20		20			
) 220 (6 *sh.*		76) 236 (3 *sh.*			
204		228			
16 Rem.		8			
12		12			
) 192 (5 *d.*		76) 106 (1 *d.*			
170		76			
22 Rem.		30			
4		4			
) 88 (2 *f.*		76) 123 (1 *f.*			
68		76			
20		47			

So that the 34th Part of *L.* 2629 is 77 : 6 : 5 : $2\frac{10}{17}$. And the 76th Part of *L.* 2747 : 16 : 10 : 3 is *L.* 36 : 3 : 1 : $1\frac{47}{76}$.

In Troy *Weight.*

lib. oz. d.wt. gr.

17) 253 : 8 : 10 : 14 (14 lib.
17

83
68

15
12
---
17) 188 (11 oz.
17

18
17

1
20
---
17) 30 (1 d.wt.
17

13
24

56
27
---
17) 326 (19 gr.
17

156
153

3

But when the Diviſor is a ſingle Figure, or the Product of two or more Figures, work as is directed, *f.* 47.

Examus, What is the 6th Part of *L.* 1076 : 18 : 4?

6) 1076 : 18 : 4
179 : 9 : 8 : $2\frac{2}{3}$ *Anſwer.*

Exa. 2. What is the 8th Part of *L.* 3419?

8) 3419
427 : 7 : 6 *Anſwer.*

Where the Remainders are reduced, and the Subtraction made mentally. See alſo the following Examples.

	L.	*s.*	*d.*	*f.*
24 } 4 } 6	1755 :	10 :	11 :	1
	438 :	17 :	8 :	$3\frac{1}{4}$
	73 :	2 :	11 :	$1\frac{21}{24}$

	L.	*s.*	*d.*	*f.*
35 } 5 } 7	197 :	18 :	6 :	3
	39 :	11 :	8 :	$2\frac{1}{4}$
	5 :	13 :	1 :	$0\frac{31}{35}$

Aver-

Aver-du-p. Weight Leſſer. Scots *Liquid Meaſure.*

	St.	lib.	oz.	dr.		hhds.	gal.	pts.	mut.	gill.
21 3	25	8	5	7	42 6	4	12	2	2	1
21 7	8	8	1	13 ø	42 7	0	12	5	3	0 ½
	1	3	7	1 $\frac{0}{7}$		0	1	6	2	0 $\frac{25}{42}$

And ſo of others.

CASE II. When Diviſor and Dividend are both applicate to the ſame kind of Things.

RULE. Reduce both to ſimple Numbers of the loweſt Denomination mentioned in either (if neceſſary) and then divide: The Quote is abſtract, and ſhews how oft the Diviſor is contained in the Dividend, or what Part it is of the ſame.

Exa. 1.	*Exa.* 2.	*Exa.* 3.
L. L	*ſh. ſh.*	*d. d.*
4) 28 (7	4) 28 (7	4) 28 (7
28	28	28

That is, 4 *L.* is the 7th Part of *L* 28, 4 *ſh.* the 7th Part of 28 *ſh.* and 4 *d.* the ſame Part of 28 *d.*

L.	*s.*	*L.*	*s.*	*d.*
38:	12)	169:	16:	8
20		20		
772		3396		
12		12		
9264	)	40760	(4	
		37056		
		3704		

So that *L.* 38: 12 is the 4th Part of *L.* 169: 16: 8, and ſomewhat more.

lib. oz. dr. ſt. oz.

What Part of 1400 : 13 : 10 is 12 : 8?

ſt. lib. oz.	lib. oz. dr.	
12 : 0 : 8)	1400 : 13 : 10	
16	16	dr. dr.
192 lib.	22413 oz.	49280) 358618 (7
16	16	344960
1160	358618 dr.	13658
192		
3080 oz.		
16		
49280 dr.		

Anſwer, the 7th Part, and ſomewhat more. And ſo of others.

REDUCTION

From a lower to an higher Species (commonly called *Reduction Aſcending*) perform'd by Diviſion.

Exa. in Money.

In 36845 *far.* how many *L. ſh.* and *d*?

4 |	36845 *f.*	Here I divide the *far.*
12 |	9211 *d.* 1 *f.*	by 4, and the Quote is
20)	76|7 *s.* 7 *d.* 1 *f.*	*d.* the *d* I divide by 12,
	38 *L.* 7 *ſh.* 7 *d.* 1 *f.*	and the Quote is *ſh.* this

laſt I divide by 20, and the Quote is *L.*

Troy *Weight.*

In 13865 gr. how many lib.

24 { 4 6 |	13865
	3466 ½
20)	57|7 : 17
12)	28 : 17 : 17
	2 : 4 : 17 : 17 *Anſr.*

Apothecary Weight.

In 13865 gr. how many lib.

20)	1386|5
3)	693 : 5
8)	231 : 0 : 5
12)	28 : 7 : 0 : 5
	2 : 4 : 7 : 0 : 5

Aver

Aver du-poiſe Weight.

In 635163 lib. how many Tuns?

28 { 4 | 635163
   { 7 | 158790 3
4) 22684 : 11
2|0) 567|1 : 0 : 11
283 : 11 : 0 : 11

Wine Meaſure.

In 76385 Pints, how many Tuns?

8) 76385
63 { 7 | 9548 : 1
   { 9 | 1364 ø
2) 151 : 3 5 : 1
2) 75 : 1 : 35 : 1
37 : 1 : 1 : 35 : 1

And ſo of others.

Queſtions to exerciſe Diviſion.

Qu. 1. What is the Value of one Yard of Cloth, when 26 yds. of the ſame coſt *L.* 2 : 18 : 6?

L. s. d.
26) 2 : 18 : 6 (0 *L.*
20
)58 (2 *ſh.*
52
6
12
)78 (3 *d.*
78

ſh. d.
Anſr. 2 : 3

Qu. 2. There is *L.* 1372 : 14 : 6 to be divided equally amongſt ſix Men, what is each's Share?

L. ſh. d.
6) 1372 : 14 : 6
228 : 15 : 9 *Anſwer.*

Qu.

Qu. 3. If I spend 186 *L.* a Year, how much is it a Week?

52) 186 (3 *L.*
156
30
20
52) 600 (11 *s.*
52
80
52
28
12
52) 336 (6 *d.*
312
24
4
52) 96 (1 *f.*
52
44

L. s. d. f.
Answ. 3 : 11 : 6 : 1 11/13

Qu. 4. How many poor People may *L.* 20 be divided amongst, so as to give Half a Crown to each?

s. d. *L.*
2 : 6) 20
12 20
30 400
12
310) 4800
160 *Answer.*

Qu.

Qu. 5. How many Tuns Burthen is that Ship of, which can carry *L.* 11000 *Sterling*, when converted into *Scots* Halfpence, each of which being $\frac{1}{4}$ of an Ounce *Aver-du-poiſe*?

11000
240 *d.* in 1 *L.*

2640000
2

4) 5280000 Halfpence or 4ths of an Ounce.
16 } 4 | 1320000 Oz.
   } 4 | 330000 ø
28 } 4 | 82500 lib.
   } 7 | 20625 ø
4) 2946 : 12 lib.
2|0) 73|6 C. 2 qrs. 12 lib.
36 T. 6 C. 2 qrs. 12 lib. *Anſwer.*

Follow ſome practical Queſtions for further Exerciſe in the foregoing Rules; to the Solution of which there is nothing further neceſſary than what has been already delivered.;

Qu. 1. I went to a Market with *L.* 150 *Sterling*, and a Horſe which coſt me 5 Guineas and a Half: I ſold the Horſe for 7 Guineas 10 *ſh.* I bought Linen Cloth to the Value of *L.* 15: 12, whereof the Seller diſcounted me a Crown. I bought a Horſe for *L.* 4: 15: 6; and ſpent of Charges before I returned 12 *ſh.* 4 *d.* As I was coming home, I had the Misfortune to drop a Purſe of 50 Guineas. How much Money remained? *Anſr. L.* 84: 12: 2

Qu. 2. I owe *L.* 137: 10: 8 to be paid preſently, whereof I have lying by me only *L.* 86: 6 *d.* how much muſt I borrow preciſely to pay this Debt?
Anſr. L. 51: 10: 2

Qu.

Qu. 3. I lent my Friend at one time *L.* 50 *Sterling*, at another 4 Guineas, at another 35 Merks and a half *Scots*; whereof he paid me at one time 2 Twenty Shillings Notes, at another he gave me 16 Yards of fine Linen at 2 *ſh. per* yd. and at another Time in Caſh *L.* 41 *Scots*. I drew a Bill on him for *L.* 39: 17: 4 *Scots* payable to *C. D.* How much does he yet owe me?

Anſr.

Qu. 4. A Butcher ſent his Servant to a Fair with *L.* 300: 12 *Scots*, to buy Sheep at *L.* 2: 5 each, Cows at *L.* 25 each, and Oxen at *L.* 30 each, of each a like Number, allowing him Half a Crown for Charges: How many of each did the foreſaid Sum purchaſe, and how much Money was returned to the Maſter?

Anſr. 5 of each, and *L.* 1: 7 *d. Sterling* returned.

Qu. 5. Suppoſe a Privateer makes a Prize to the Value of *L.* 3578: 14: 6, how much is due to each of the Crew (being 47 in all) allowing the Captain $\frac{1}{10}$, and the other Officers (who are 4 in Number) each $\frac{1}{25}$ Part of the Remainder, and what remains over and above being to be equally diſtributed amongſt the private Men?

Anſr.
Captain's Share, *L.* 223: 13: 4: $3\frac{1}{2}$
Each Officer's Share, 104: 16: 10: $3\frac{4}{84}$
Each private Man's Share, 69: 17: 11: $1\frac{3}{3}$

Qu. 6. Suppoſe I have ſpent of Caſh theſe 8 Years laſt paſt *L.* 838: 11, and have run in debt to the Sum of *L.* 75: 19 over and above, how much comes it to *per* Year, *per* Month, *per* Week and *per* Day?

Anſr.
L. 114: 6: 3 *per Ann.*
8: 15: $4\frac{44}{73}$ *per* Mon.
2: 3: $10\frac{11}{73}$ *per* Week.
6: $3\frac{12}{73}$ *per* Day.

Qu. 7. Suppoſe I ſpent laſt Year *L.* 114: 6: 3, how much will ſerve me $20\frac{1}{2}$ Years at the ſame Rate?

Anſr. L. 2343: 8: $1\frac{1}{2}$

Qu.

Qu. 8. Suppoſe I have $\frac{1}{16}$ of a Ship, and her Freight for 3 Voyages is *L.* 345 *Sterling*, how much of the ſame falls to me, there being *L.* 13: 11 to be paid by the Partners for Charges on the ſaid Voyages?

Anſr. *L.* 20: 14: $3\frac{3}{4}$

Qu. 9. I went to a Market with *L.* 60: 5, where I bought 400 yds. of Linen at $10\frac{1}{2}$ *d. per* yd. alſo 250 yds. of Plaiden at 5 *d. per* yd. alſo $4\frac{1}{2}$ Groſs coarſe Stockings at 13 *ſh.* 4 *d. per* Doz. I ſpent of Charges 7 *ſh.* 6 *d.* When I returned home I had preciſely *L.* 1: 2: 4. whether ought I to have had more or leſs? *Anſr.* I ſhould have had a Shilling more.

Qu. 10. I bought 3 Laſt of Salmons at *L.* 2: 5 *per* Bar. which I ſent abroad. I ſpent of Charges at Shipping for Freight, *&c.* *L.* 2: 10. My Factor adviſed me that he had ſold the ſame for ready Money, the neat Proceeds whereof (all Charges deducted) amounted to *L.* 103: 10, for which he ſent me in return 414 lib. of Tea. How much was gained by the Salmons, and what did the Tea coſt me *per* lib. as alſo how may I ſell the ſame to gain *L.* 32: 1: 6 on the Salmons?

Anſr. { *L.* 20 Gain on the Salmons.
5 *ſh.* prime Coſt of the Tea *per* lib.
5 *ſh.* 7 *d.* Price of the Tea at the propoſed Gain.

Qu. 11. Of what Quantity is 37 Bolls 12 Pecks the 17th Part? Ch. B. pecks.

Anſr. 40: 1: 12

Qu. 12. A Merchant has 4 hhds. of Wine, containing each (ſuppoſe) 130 Pints *Scots*, which he deſigns to draw off in Chopin Bottles; how many Groſs will he need for this purpoſe? Gr. doz. bottles.

Anſr. 7: 2: 8

Qu. 13. I lent my Friend *L.* 56: 18 *Sterl.* in return of which he gave me 45 Piſtoles, valued each at 17 *ſh.*

6 *d.* and 15 Carolus's of Gold at 23 *ſh.* each; was I fully paid? *Anſr.* He ſtill owed me 5 *ſh.* 6 *d.*

Qu. 14. How many ſquare Stones of 10 Inches, and $1\frac{1}{2}$ Inch thick, will pave a Floor which is 5 yds. long, and half as broad? *Anſr.* 162.

Qu. 15. I have to ſell $4\frac{1}{2}$ doz. fine Stockings, a Merchant has offer'd me 5 *ſh.* 6 *d. per* Pair, and to pay for the marking of them: Another tells me he'll give me *L.* 3: 6 *per* doz. without being at the Charge of the Stamp; and a Third offers me 15 Stone of Wool worth 15 *d. per* lib. I being at the Charge of the Stamping. Which of all theſe is the beſt Bargain, and by how much does it exceed the other two?

Anſr. The laſt is the beſt Bargain, *viz.* better than the firſt by 1 *s.* 6 *d.* and better than the other by 3 *ſh.* (I paying the Stamp.)

Note. The Stamp is 2 *d. Scots*, or $\frac{1}{6}$ *d. Sterling per* Pair.

Qu. 16. How much *Sterling* Money is contained in *L.* 1874: 10: 6 *Scots*?

Anſr. L. 156: 4: $2\frac{1}{2}$, found by dividing by 12.

Qu. 17. In *L.* 156: 4: $2\frac{1}{2}$ *Sterling*, how much *Scots* Money?

Anſr. 1874: 10: 6, found by multiplying by 12.

Qu. 18. In 4176 *L.* how many Merks?

Anſr. 6264 Merks, found by multiplying by 3, and dividing the Product by 2; or by adding $\frac{1}{2}$ of 4176 to itſelf.

Qu. 19. In 3126 Merks how many *L.*?

Anſr. L. 2084, found by multiplying by 2, and dividing the Product by 3; or by ſubtracting $\frac{1}{3}$ of 3126 from itſelf.

Qu. 20. In 20600 Merks *Scots* how much *Sterling* Money?

Anſr. L. 1144: 8: $10\frac{2}{3}$, found by dividing by 18.

Qu.

Qu. 21. In *L.* 145 : 14 : 10 *Sterl.* how many *Scots* Merks?

Anſr. 2623 Merks, and 4 $\frac{2}{3}$ *d. Sterl.* over and above.

Qu. 22. In 7000 Guilders at 23 $\frac{1}{2}$ *per* Guilder, how much *Sterling* and how much *Scots* Money?

Anſr. *L.* 685 : 8 : 4 *Sterling*, and
L. 8225 : 0 : 0 *Scots.*

Qu. 23. In 436 Guineas at 21 *ſh.* how many *L*?

Anſr. *L.* 457 : 16, found by multiplying by 21, and dividing the Product by 20; or by adding $\frac{1}{20}$ of 436 to itſelf.

CHAP. VI. *Of Arithmetical Progreſſion.*

WHEN a Rank of Numbers do either increaſe or decreaſe by a common Difference, thoſe Numbers are ſaid to be in Arithmetical Progreſſion: Thus,

{1. 2. 3. 4. 5. 6. 7. *&c.* / 7. 6. 5. 4. 3. 2. 1.} Here the com. Difference is 1.

{1. 3. 5. 7. 9. 11. 13. *&c.* / 13. 11. 9. 7. 5. 3. 1.} Here the com. Difference is 2.

And ſo of any other Series whoſe Difference is 3, 4, 5, *&c.*

In every Arith. Progreſſion there are 5 Things conſiderable, *viz.*

The two Extremes, *i. e.* the firſt and laſt Term of the Series.

The Common Difference.

The Number of Terms. And

The Sum of all the Series.

And the moſt uſeful Caſes are theſe, wherein are 3 given, to find the other 2.

CASE 1. Having the Extremes, and Number of Terms, to find the common Difference and Sum of all the Series.

RULE. Divide the Difference of the Extremes by the Number of Terms less 1, and the Quote is the Difference: And multiply the Sum of the Extremes by the Number of Terms, and half this Product is the Sum of the Series.

Exa. A Gentleman laid by for a younger Son's Portion on the first Day of *January* L. 12, and continued to lay by a certain Sum in Arith. Progression on the first Day of each Month to the End of the Year, the last Sum being L. 45. How much did each exceed the former, and what did his Portion amount to in all?

45—12=33 Difference of Extremes.
12—1=11 Number of Terms less 1.
Then 11) 33 (3 common Difference or Excess.
And 12+45=57 Sum of the Extremes.
12 Number of Terms.

2) 684 (342 his Portion, or Sum of the Terms.

Exa. 2. Suppose 100 Eggs were placed in a right Line, a Yard distant from each other, and the first Egg a Yard distant from a Basket; how many Yards will one walk (beginning at the Basket) before he can gather up those 100 Eggs singly one after another, returning with each to the Basket, and putting them in without breaking?

2+200=202 Sum of the Extremes.
100 Number of Terms.

2)20200
10100 Number of Yards, *Answer.*
And 1760) 10100 (5 Miles and 3 Quarters *ferè.*

CASE

CASE 2. Having the Extremes and common Difference, to find the Number of Terms and Sum of the Series.

RULE. Divide the Difference of the Extremes by the common Difference, and to the Quote adding 1, the Sum is the Number of Terms: then find the Sum of the Series by Caſe 1ſt.

Exa. A Gentleman laid by for a younger Son's Portion on the firſt Day of *January* L. 12, and continued to lay by a Sum on the firſt Day of each Month for a certain Number, increaſing each by L. 3, the laſt wherof was 45 L. How many Months did he continue ſo to do, and what did his Portion amount to in all?

45—12=33 Difference of Extremes.
3)33(11, and 11+1=12 Number of Terms.
Then 12+45=57 Sum of the Extremes.
12
2)684(342 L. Sum of the Series.

CASE 3. Having one of the Extremes, the Number of Terms and the common Difference, to find the other Extreme and the Sum of all the Terms.

RULE. If it is the leſſer Extreme that's given, multiply the common Difference by the Number of Terms leſs 1, and to the Product add the given Extreme; but if it is the greater Extreme that's given, ſubtract it, and the Sum or Remainder is the other Extreme, and find the Sum of all the Terms by Caſe 1 or 2.

Exa. A Gentleman laid by for a younger Son's Portion on the firſt of *January* L. 12, on the firſt of *February* L. 15, and ſo on to the End of the Year, increaſing each by L. 3. How much did he lay by on the firſt of *December*, and what was his Portion?

3 common Difference.
11 Number of Terms leſs 1.

33+12=45 laid by on the firſt of *December.*
Then 12+45=57 Sum of the Extremes.
12 Number of Terms.

2)684 (342 Sum of all the Terms.

CASE 4. Having the Extremes; and Sum of all the Series, to find the Number of Terms and common Difference.

RULE. Divide double the Sum by the Sum of the Extremes, the Quote is the Number of Terms: then find the common Difference by one of the preceding Caſes.

Exa. A Gentleman laid by for a younger Son's Portion on the firſt of *January L.* 12, and continued ſo to do on each firſt Day of the ſucceeding Months for a certain Number, increaſing each by Arithmetical Progreſſion; ſo that the laſt Sum laid by was *L.* 45. How many Months did he continue to lay by, and by how much did each Sum exceed another, the Sum of all he laid by being *L.* 342?

342 Sum of all the Series.
2 12+45=57 Sum of Extr.

57)684(12: Number of Terms.

Then by Caſe 1ſt to find the common Difference. 45—12=33 Diff. of Extremes.; 12—1=11 Number of Terms leſs 1. Therefore 11)33(3 common Difference.

CASE 5. Having one Extreme, Sum of the Series and Number of Terms, to find the other Extreme and the common Difference.

RULE. Divide double the Sum of the Series by the Number of Terms, and from the Quote ſubtract the

the given Extreme, the Remainder is the Extreme ſought; then find the Difference by one of the preceding Caſes.

Exa. A Gentleman laid by for a younger Son's Portion on the firſt Day of *January* L. 12, and on the firſt Day of the ſucceeding Months of the Year a certain Sum, each increaſing in Arithmetical Progreſſion, ſo that he laid by in all L. 342. What was the Sum he laid by on the firſt of *December*, and by how much did each Sum exceed another?

342
2
—
12) 684 (57 Sum of the Extremes.
12
—
45 laid by on the firſt of *December*.

Then by laſt Caſe to find the common Difference. 45—12=33; and 12—1=11. Then 11)33(3 com. Diff.

CASE 6. Having the Sum of the Series, common Difference and Number of Terms, to find the Extremes.

RULE. By the Sum and Number of Terms find the Sum of the Extremes, as in laſt Examp. then, by means of the common Difference and Number of Terms, find the Difference of the Extremes thus, Multiply the common Difference by the Number of Terms leſs 1, and the Product is the Difference of the Extremes. Laſtly, having the Sum and Difference of the Extremes, find the Extremes thus, half their Sum + half their Difference is the greater, and half their Difference ſubtracted from half their Sum is the leſſer.

Exa. A Gentleman laid by for a younger Son's Portion on the firſt Day of *January* a certain Sum, and ſo continued to do on the firſt Day of each ſucceeding Month to the End of the Year, increaſing

each by *L.* 3. ſo that the Amount of all came to *L.* 342: How much did he lay by on the firſt Day of *Jan.* and alſo on the firſt Day of *December*?

342
2

12) 684 (57 Sum of the Extremes.
3 comm. Diff. 12—1=11 Number of Terms leſs 1.
11

33 Difference of Extremes.
2) 57 (28 ½ half the Sum of the Extremes.
2) 33 (16 ½ half the Diff. of the Extremes.

Sum 45 greater Extreme, and 57—45=12 for the leſſer Extreme.

CASE 7. Having one Extreme, Sum of the Series and common Difference, to find the other Extreme and Number of Terms.

RULE. If the leſſer Extreme is required, ſquare the greater Extreme, to which add its Product by the common Difference, + the 4th Part of the Square of the common Difference, from which Total ſubtract the Product of twice the Difference by the Sum of the Series; out of which Remainder extracting the ſquare Root, to it add the common Difference, and the Sum is the leſſer Extreme. But, if the greater Extreme is required, To the Square of the leſſer add the Product of the Sum of the Series into twice the common Difference, and to this Sum add the 4th Part of the Square of the common Difference, from which Sum ſubtracting the Product of the given Extreme by the common Difference, take the ſquare Root of the Remainder, and from this Remainder ſubtract half of the common Difference, and the Remainder is the greater Extreme.

CHAP. VII. *Of Geometrical Progreſſion.*

WHEN a Rank of Numbers do either increaſe by a common Multiplicator, or decreaſe by a common Diviſor, theſe Numbers are ſaid to be in Geometrical Progreſſion: As

{2. 4. 8. 16. 32. *&c.* where 2 is the com. Multiplier.
{64. 32. 16. 8. 4. *&c.* where 2 is the com. Diviſor.

Or {2. 6. 18. 54. 162, *&c.* {where 3 is the com. Multiplier.
{162. 54. 18. 6. 2. where 3 is the com. Diviſor.

In Geometrical Progreſſion (as in Arithmetical) there are 5 Things conſiderable, *viz.*

The 2 Extremes.

The *Ratio*, *i. e.* the Quote of the greater Term divided by the leſſer, or the common Multiplier.

The Number of Terms.

The Sum of the whole Series.

And the moſt uſeful Caſes are thoſe, in which are given any of theſe 3 Things, to find the other 2.

CASE 1. Having the Extremes and Ratio, to find the Sum and Number of Terms.

RULE. For the Sum of the Series, divide the Difference of the Extremes by the Ratio leſs 1, the Quote is the Sum leſs the greater Extreme. And to find the Number of Terms, divide the greater Extreme by the leſſer, and raiſe the Ratio to a Power equal to that Quote, and the Index or Number of Multiplications is the Number of Terms leſs 1.

CASE 2. Having the Extremes and Number of Terms, to find the Sum and Ratio.

RULE

RULE

1. For the Ratio, divide the greater Extreme by the leſſer, extract ſuch a Root of the Quote whoſe Index is the Number of Terms leſs 1, and it is the Ratio: And find the Sum by Caſe 1.

CASE 3. Having the Extremes and Sum, to find the Ratio and Number of Terms.

RULE

1. For the Ratio, divide the Difference of the Sum and leſſer Extreme, by the Difference of the Sum and greater Extreme; and the Quote is the Ratio. And for the Number of Terms, find it by Caſe 1.

CASE 4. Having either of the Extremes, the Ratio and Number of Terms, to find the other Extreme, and the Sum of the Series.

RULE. If it is the leſſer Extreme that's given, multiply it by the Ratio multiplied into itſelf as oft as is the Number of Terms leſs 1, and the Product is the greater Extreme.

But if the greater Extreme is given, divide it by the Ratio multiplied into itſelf as oft as is the Number of Terms leſs 1, and the Quote is the leſſer Extreme; And for the Sum of the Series, find it by Caſe 1.

CASE 5. Having the Sum of the Series, Number of Terms and Ratio, to find the Extremes.

RULE. To find the leſſer Extreme, multiply the Ratio as oft into itſelf as is the Number of Terms; then multiply the Sum of the Series by the Ratio leſs 1, and divide the Product by the foreſaid Power of the Ratio leſs 1, the Quote is the leſſer Extreme; and for the other Extreme, find it by laſt Caſe.

CASE 6. Having either Extreme, the Sum and Ratio, to find the other Extreme and Number of Terms.

RULE.

RULE. If the greater Extreme is given, to find the leſſer, multiply the Difference of the Sum and greater Extreme by the Ratio leſs 1, ſubtract the Product from the greater Extreme, and the Remainder is the leſſer: And find the Number of Terms by Caſe 1.

But, if the leſſer Extreme is given, to find the greater, multiply the Difference of the Sum and leſſer Extreme by the Ratio leſs 1, and divide the Product by the Ratio, and to the Quote adding the leſſer Extreme, the Sum is the greater; finding the Number of Terms as before.

Now to exemplify ſome of the preceding Caſes, take the following Queſtion.

A Servant, skill'd in Numbers, agreed with a Gentleman to ſerve him for 12 Months, provided he would give him a Farthing for his firſt Month's Service, a Penny for the ſecond, 4 Pence for the third, and ſo on, quadrupling each preceding Month's Wages, to the end of the Year: What did his Hire amount to?

By Caſe 4. given the leſſer Extreme 1 *far.*

And 4 rais'd to the 11th Power is 4194304 for the greater Extreme, or laſt Month's Wages.

Then 4194304—1=4194303 Differ. of Extremes.
Alſo 4—1=3=Ratio leſs 1, and 3)4194303(1398101

Laſtly, 1398101+4194304=5592405 *far*=
L. 5825 : 8 : 5 : 1 *Anſwer.*

Which Queſtion you may vary like that in Arith. Progreſſion, and ſo apply the other Caſes.

CHAP.

CHAP. VIII. *Of* FRACTIONS.

THERE are only two Kinds of Fractions now generally uſed, *viz. Vulgar* or *Common*, and *Decimal*.

A Common Fraction conſiſts of two Members, *viz.* a Numerator and a Denominator, which are written as in the following Example $\frac{3}{4}$ or $3/4$, the Figure 3 being the Numerator, and 4 the Denominator. The firſt Form ſeems moſt convenient for Calculations, and the other for keeping Accompts.

The Denominator ſhews into how many Parts the Integer or whole Thing is divided, or ſuppoſed to be divided.

The Numerator ſhews how many of theſe Parts are contained in the given Fraction: Thus in the adduced Example $\frac{3}{4}$, the Denominator ſhews, that the whole Thing (whatever it is) is divided, or ſuppoſed to be divided into 4 equal Parts; and the Numerator ſhews, that 3 of theſe equal Parts are contained in the Fraction. In like manner,

$\left.\begin{matrix}\frac{1}{2}\\ \frac{2}{3}\\ \frac{4}{5}\\ \frac{9}{16}\end{matrix}\right\}$ ſignifies $\begin{cases}\text{One half.}\\ \text{Two third Parts, or 2 Thirds.}\\ \text{4 fifth Parts, or 4 Fifths.}\\ \text{9 ſixteenth Parts, or 9 Sixteenths.}\end{cases}$

Or we may read them thus: 1 divided by 2; 2 divided by 3; 4 divided by 5; 9 divided by 16; and ſo of others: For the Numerator is always the Dividend, and the Denominator the Diviſor.

A Common Fraction is Proper or Improper, Simple or Compound.

A Proper Common Fraction is ſuch whoſe Numerator is leſs than its Denominator, and conſequently the Fraction is leſs than the Integer, to which it refers. *Exa.* $\frac{1}{2}$, $\frac{6}{7}$, $\frac{15}{18}$, *&c.*

An

An Improper Common Fraction is ſuch whoſe Numerator is equal to or greater than its Denominator, and therefore the Fraction is equal to or greater than its relative Integer. *Exa.* $\frac{3}{3}$, $\frac{9}{8}$, $\frac{10}{7}$, $\frac{24}{6}$, $\frac{41}{14}$, *&c.* And here note, that if the Numerator and Denominator, are both the ſame, the Fraction is equal to the Integer, as $\frac{8}{8}$, $\frac{6}{6}$, $\frac{12}{12}$, $\frac{45}{45}$, *&c.*

A Simple Common Fraction has only one Numerator and one Denominator, whether it be Proper or Improper. *Exa.* $\frac{3}{8}$, $\frac{9}{7}$, $\frac{11}{18}$, $\frac{8}{8}$, $\frac{10}{7}$, *&c.*

But a Compound Fraction is a Fraction of a Fraction, and hath ſeveral Numerators and ſeveral Denominators; that is, it conſiſts of ſeveral Simple Fractions, the Particle *of* ſtanding betwixt each two of them. *Exa.* $\frac{4}{5}$ of $\frac{2}{3}$; by which is meant, firſt, that the whole Thing or Integer is divided into 3 equal Parts, two of which Parts make $\frac{2}{3}$; then, that this Fraction $\frac{2}{3}$ is divided into 5 equal Parts, and 4 of theſe Parts taken. In like manner $\frac{1}{6}$ of $\frac{1}{8}$ of $\frac{9}{13}$, $\frac{2}{7}$ of $\frac{3}{4}$ of $\frac{1}{2}$ of $\frac{1}{10}$, are Compound Fractions.

A Mixt Number (in Fractions) is a whole Number with a Fraction annex'd, as $4\frac{1}{2}$, $20\frac{3}{4}$, which are read four and one half, 20 and 3 fourth Parts, where the Fractions are ſuppoſed to refer to the ſame Integers with the 4 and 20 reſpectively; ſo that if $4\frac{1}{2}$ be $4\frac{1}{2}$ *L.* its Value is *L.* 4: 10; and if the $20\frac{3}{4}$ be $20\frac{3}{4}$ yds, the Value is 20 yds and 3 Quarters.

CHAP. IX. *Of Reduction of Common Fractions.*

HERE I would adviſe the Learner to make himſelf Maſter of the following XI. Caſes, becauſe upon the

the right underſtanding of them depends the moſt of what is to be delivered in Addition, Subtraction, &c. and they are as it were the very Hinge upon which (almoſt) all the Operations, where Fractions are concern'd, do turn.

CASE I.

To reduce a Fraction to its loweſt Term, that is, to find another equivalent Fraction to a given one, but expreſs'd in lower Terms (when poſſible.)

RULE.

Divide the greater Term by the leſſer, that is, the Numerator or Denominator, the one by the other; and if there is a Remainder, by it divide the firſt Diviſor; and if any thing yet remains, let it be a Diviſor to the laſt Diviſor, and thus go on, by ſtill dividing the preceding Diviſor by the laſt Remainder, till 0 remain: the ſeveral Quotes are to be neglected, and the laſt Diviſor is to be aſſumed as the greateſt common Meaſure to the Terms of the propoſed Fraction, by which common Meaſure dividing the Numerator and Denominator, ſeverally, you'll have the Fraction expreſs'd in its loweſt equivalent Terms.

Exa. 1. Reduce $\frac{16}{56}$ to its loweſt Terms.

Operation.

16) 56 (3
48
—
8) 16 (2
16
—
0

Or thus,

3
—
16 | 2
—
8

Their greateſt common Meaſure is 8, wherefore by dividing both Terms by it, (thus, 8) $\frac{16}{56}$ ($\frac{2}{7}$) the Fraction $\frac{16}{56}$ is reduced to $\frac{2}{7}$, its loweſt equivalent Expreſſion.

Exa.

Exa. 2. Reduce $\frac{2832}{12848}$ to its leaſt Terms.

Operation.

2832) 12848 (4
11328
1520) 2832 (1
1520
1312) 1520 (1
1312
208) 1312 (6
1248
64) 208 (3
192
16) 64 (4
64

Or thus,

12848	4
2832	1
1520	1
1312	6
208	3
64	4
16	
0	

Their greateſt common Meaſure (thus found) is 16, wherefore $2832 \div 16 = 177$ Numerator, and $12848 \div 16 = 803$ Denominator, and ſo $\frac{2832}{12848}$ in its loweſt Terms is $\frac{177}{803}$.

Exa. 3. Reduce $3\frac{384}{28}$ to its loweſt Expreſſion.

28) 384 (13
28
104
84
20) 28 (1
20
8) 20 (2
16
4) 8 (2
8

Or thus,

384	13
28	1
104	
20	2
8	2
4	
0	

Then 4) $3\frac{84}{28}$ ($9\frac{6}{7}$ *Answer* in its lowest Terms.

1. *Observe*, If Unity is the last Divisor, the Fraction is irreducible, and is already in its lowest Terms; as in the Exa. following, $1\frac{54}{97}$.

Operation.

54) 197 (3
162
35) 54 (1
35
19) 35 (1
19
16) 19 (1
16
3) 16 (5
15
1) 3 (3
3
0

Or shorter thus,

197	3
54	1
35	1
19	1
16	5
3	3
1	
0	

2. *Observe*, That the general Rule is not so convenient for Practice as the taking $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, *&c.* (or dividing by 2, 3, 4, *&c.*) of both Numerator and Denominator, (as you see practicable) to divide them without a Remainder. Such Divisors, after a little Practice and Experience, will discover themselves almost at first view, without any previous Trial.

Exa. $\frac{294}{336}$ in its lowest Terms is $\frac{7}{8}$, found thus:

$$3\left|\frac{294}{336}\right|\frac{7}{112}\left|\frac{2}{16}\right|\frac{14}{7}\left|\frac{7}{8}\right.$$ *Answer.*

3. *Observe*, If Numerator and Denominator end in an even Number, they may both be divided by 2, without any Remainder; and if they are both even after the first Division, you may divide again the Quote by

by 2, and ſo continue to divide by 2 as oft as poſſible, and after that by any Figure that will do it, till you come to its loweſt Terms.

Exa. $\frac{8}{18}$ is $\frac{4}{9}$; $\frac{192}{288}$ is $\frac{2}{3}$, thus found,

$$2)\frac{192}{288}\Big|\frac{^{2}96}{144}\Big|\frac{^{2}48}{72}\Big|\frac{^{2}24}{36}\Big|\frac{^{2}12}{18}\Big|\frac{^{3}6}{9}\Big|\frac{2}{3}$$

Here I divide 5 times by 2, and once by 3. Now tho' 2 will certainly divide every even Number, yet you'll frequently at firſt Sight obſerve, that ſome greater Number will do it, which you ought to take, becauſe the greater the Diviſors are, the fewer there muſt be of them, and the Operation is the ſooner ended. So in the laſt Exa. to have divided by 8 and 12 would have produced the ſame Effect.

4. *Obſerve,* If both Numerator and Denominator, end in 0 or 0's, cut off an equal Number from both, and you have the Fraction lower, which when practicable you may reduce, till you come to the loweſt.

Exa. $\frac{400}{500}$ is $\frac{4}{5}$; $\frac{2700}{6300}$ is firſt $\frac{27}{63}$, and by dividing by 9, it becomes $\frac{3}{7}$ loweſt. This Abbreviation is the ſame with dividing both by 10, 100, 1000, *&c.*

5. *Obſerve,* If Numerator and Denominator, the one of them end in 0 and the other in 5, they are both diviſible by 5, without a Remainder. *Exa.* $\frac{5}{40}=\frac{1}{8}$; $\frac{20}{45}=\frac{4}{9}$. Alſo if both end in 5, thus $\frac{15}{25}=\frac{3}{5}$, and $\frac{175}{185}=\frac{15}{37}$.

USE. This Reduction is neceſſary for facilitating any Operation with them: Alſo a Fraction muſt be reduced to its loweſt Terms, before its Root can be extracted.

CASE II.

To reduce an Improper Fraction to its equivalent whole or mixt Number.

RULE. Divide the Numerator by the Denominator, the Quote is the whole Number; and if there is a Remainder, it muſt be annexed to the Integral Part in form of a Fraction, and ſo the Anſwer will be a

mixt Number. *Exa.* $\frac{3}{1}=3$; $\frac{12}{3}=4$; $\frac{17}{5}=3\frac{2}{5}$; $\frac{25}{7}=3\frac{4}{7}$; $\frac{28}{6}=4\frac{2}{3}$.

USE. This Reduction is of use for the better understanding their Value.

CASE III.

To reduce a whole Number to an equivalent (Improper) Fraction, having any assigned Denominator.

RULE. Multiply the whole Number by the assigned Denominator, the Product is the Numerator, which set over the assigned Denominator, gives the Answer. *Exa.* Reduce 6 to an Improper Fraction, whose Denominator is 8. Thus, $6\times8=48$; then $\frac{48}{8}$ is the Thing sought: This is the Converse of the last Case.

USE. This is of use when a Fraction is to be added to or subtracted from a whole Number, *viz.* that the Sum or Remainder may be in form of a Fraction.

CASE IV.

To reduce a whole Number to an equivalent (Improper) Fraction, when no Denominator is given.

RULE. Set the whole Number for a Numerator, and 1 for a Denominator, and it is done. *Exa.* 4 is $\frac{4}{1}$; 12 is $\frac{12}{1}$.

USE. Such Reduction is necessary when a whole Number is to be multiplied or divided by a Fraction.

CASE V.

To reduce a mixt Number to an equivalent Improper (simple) Fraction.

RULE. Multiply the whole Number by the Denominator of the annexed Fraction, adding to the Product its Numerator, the Sum is the Numerator, and the Denominator given is the Denominator of the Answer. *Exa.* Reduce $3\frac{2}{5}$ to a simple Fraction; thus, $3\times5=15$, and $15+2=17$; so the Answer is $\frac{17}{5}$. *Exa.* 2. Reduce $16\frac{3}{7}$ to a simple Fraction; thus, $16\times7=112$, and $112+3=115$; so the Answer is $\frac{115}{7}$.

USE.

USE. This Reduction is neceſſary, when a mixt Number is to be added to, multiplied or divided by a Fraction; or when a Fraction of a different Denominator, is to be ſubtracted from it, or it from another mixt Number of a different Denominator.

CASE VI.

To reduce a Compound Fraction to a Simple equivalent one.

RULE. Multiply all the Numerators continually into one another for the Numerator, and all the Denominators together for the Denominator of the Anſwer, which you may reduce to its loweſt, if it is not ſo after the Multiplication. *Exa.* Reduce $\frac{4}{5}$ of $\frac{2}{3}$ to a Simple Fraction; thus, $4 \times 2 = 8$ Numerator, and $5 \times 3 = 15$ Denominator; therefore $\frac{8}{15}$ is the Anſwer. Alſo $\frac{3}{4}$ of $\frac{6}{7}$ of $\frac{1}{8} = \frac{18}{56}$ for the Simple equivalent Fraction, which in its loweſt Terms is $\frac{9}{28}$.

1. If one of the Members of a Compound Fraction is a whole Number greater than 1, then this whole Number being turned into an Improper Fraction (by Caſe 4th), let the Reduction be made as before. *Exa.* $\frac{2}{9}$ of 4 is $\frac{2}{9}$ of $\frac{4}{1} = \frac{8}{9}$; $\frac{3}{4}$ of 8 is $\frac{3}{4}$ of $\frac{8}{1} = {}^{2}\frac{24}{4} = 6$; $\frac{2}{3}$ of $17 = \frac{2}{3}$ of ${}^{17}\frac{17}{1} = {}^{3}\frac{34}{3} = 11\frac{1}{3}$.

2. When one of the Members is a mixt Number, reduce it by Caſe 5th to a Simple Fraction, and work as before. *Exa.* $\frac{3}{5}$ of $4\frac{1}{2}$ is $\frac{3}{5}$ of $\frac{9}{2} = \frac{27}{10} = 2\frac{7}{10}$.

3. When all or any one of the given Fractions can be reduced to lower Terms, let that be done firſt, and then apply the general Rule; for by this Means the Anſwer will come out in lower Terms, tho' not always in its loweſt. *Exa.* $\frac{4}{6}$ of $\frac{3}{4}$ is $\frac{3}{3}$ of $\frac{3}{4} = \frac{9}{12}$ or $\frac{3}{4}$; $\frac{2}{4}$ of $\frac{6}{3}$ is $\frac{1}{2}$ of $\frac{2}{1} = 1$.

4. It matters not in what order the Members of a Compound Fraction are taken, for $\frac{2}{3}$ of $\frac{1}{2}$ is the ſame with $\frac{1}{2}$ of $\frac{2}{3}$.

USE. This Reduction is of uſe for the more ready comprehending their Value, as alſo in order to prepare them for Addition, Subtraction, Multiplication and Diviſion.

CASE VII.

To reduce Fractions of different Denominators to other equivalent Fractions having the ſame Denominator.

RULE. Multiply all the Denominators continually into one another, the Product is the common Denominator ſought; then multiply the Numerator of each Fraction into the Denominators of all the other continually, and the Product of each is a new Numerator. *Exa.* Reduce $\frac{2}{3}$ and $\frac{5}{8}$ to the ſame Denominator: Thus $3 \times 8 = 24$ their common Denominator; then $2 \times 8 = 16$ firſt Numerator, and $5 \times 3 = 15$ the other Numerator; ſo the equivalent Fractions found are $\frac{16}{24}$ and $\frac{15}{24}$; for $\frac{2}{3}$ is $= \frac{16}{24}$, and $\frac{5}{8}$ is $= \frac{15}{24}$: The Denominator may be ſet down only once, thus, $\frac{16\ 15}{24}$. Reduce $\frac{4}{7}$, $\frac{13}{18}$ and $\frac{2}{5}$ to a common Denominator; thus, $7 \times 18 = 126$, and $126 \times 5 = 630$ for the Denominator; then $4 \times 18 \times 5 = 360$ firſt Numerator; next $13 \times 7 \times 5 = 455$ the ſecond Numerator; laſtly $2 \times 18 \times 7 = 252$ the laſt Numerator. So the equivalent Fractions found are $\frac{360}{630} = \frac{4}{7}$, $\frac{455}{630} = \frac{13}{18}$, and $\frac{252}{630} = \frac{2}{5}$, and may ſtand thus, $\frac{360\ 455\ 252}{630}$.

1. After the common Denominator is found, you may find the new Numerator thus: Divide the common Denominator by the Denominator of each of the Fractions, and multiply the Quotes by the Numerators reſpectively, and ſo you have the new Numerators ſought.

2. When there are two Fractions to be reduced, and one of the Denominators is a Multiple of the other, divide; and by the Quote multiply the Numerator and Denominator of that Fraction whoſe Denominator you divide by, and thus they'll be both reduced to the ſame Denominator. *Exa.* To reduce $\frac{3}{4}$ and $\frac{5}{8}$ to the ſame Denominator, I divide the Denominator 8 by 4 the Denominator of the other Fraction, and the Quote is 2; by which multiplying the Fraction $\frac{3}{4}$, it is reduced to $\frac{6}{8}$, having the ſame Denomina-

tor,

tor with the other Fraction. The Advantage of this Method is, that it gives the Fractions frequently, tho' not always, in their loweſt Terms.

USE. This Reduction is neceſſary before they can be added or ſubtracted, as alſo to know which of two Fractions propoſed is the greater (if they do not diſcover themſelves at firſt ſight.) It is further uſeful for finding two Integers in the ſame Proportion to one another with the Fractions given: For inſtance, to find two Integers in proportion to one another, as $\frac{2}{3}$ is to $\frac{3}{4}$; thus, $\frac{2}{3} : \frac{3}{4} :: \frac{8}{12} : \frac{9}{12}$, that is, as 8 to 9; ſo that 8 and 9 are the Integers ſought.

CASE VIII.

To reduce a Fraction of an Unit of an higher Denomination, or Name, to an equivalent Fraction of any Unit of a lower Species (of the ſame Kind with the higher.)

RULE. Multiply the Numerator of the given Fraction by the Number of Units in the next inferior Species that makes an Unit of the Denomination of your Fraction, and the Product multiply by the Number of Units in the next inferior Denomination that make an Unit of the laſt Denomination, and thus proceed 'till you come to the loweſt you deſign, then make the laſt Product a Numerator to the Denominator of the Fraction given. *Exa.* Reduce $\frac{3}{5}$ *L.* to an equivalent Fraction in the Denomination of 1 *d.*

3 Numerator of the given Fract.
20 Number of *ſh.* in 1 *L.*

60
12 Number of *d.* in 1 *ſh.*

720

So that $\frac{3}{5}$ *L.* is $= \frac{720}{5}$ *d.* or $\frac{144}{1}$ *d* loweſt.

Exa.

Exa. 2. Reduce $\frac{1}{7}$ *ſh.* to the Fraction of 1 *farth.*

1 the Numerator given.
12 : Number of *d.* in 1 *ſh.*

12

4 : Number of *far.* in 1 *d.*

48

So that $\frac{1}{7}$ *ſh.* is $= 48\frac{3}{7}$ *far.*

Exa. 3. Reduce $\frac{4}{7}$ C. to the Fraction of 1 oz.

2
4

8 qrs.
28

224 lib.
16

3584 oz.

So that $\frac{4}{7}$ C. is equal to $\frac{3584}{3}$ oz.

Or you may expreſs them in form of Compound Fractions, and then reduce them to Simple ones; ſo in the firſt Exa. $\frac{3}{5}$ *L.* is $\frac{3}{5}$ of 20 *s.* of 12 *d.* $= \frac{3}{5}$ of $\frac{20}{1}$ of $\frac{12}{1} = \frac{720}{5}$, as before. In Exa. 3. $\frac{2}{3}$ C. is $\frac{2}{3}$ of 4 qrs. of 28 lib. of 16 oz. $= \frac{2}{3}$ of $\frac{4}{1}$ of $\frac{28}{1}$ of $\frac{16}{1} = \frac{3584}{3}$ oz.

USE. The Uſe of this Reduction will appear in that of the next following Caſe.

CASE IX.

To reduce a Fraction of an Unit of a lower Denomination, to an equivalent Fraction in the Denomination of an higher.

RULE. Multiply the Denominator of the propoſed Fraction by the Number of Units of the ſaid Fraction that is equal to an Unit of the next ſuperiour Denomination, and the Product by ſuch a Number of Units of its Denomination, as is equal to an Unit of the next above it; and thus go on till you come to the higheſt

higheſt Species required, and the laſt Product is a Denominator to the Numerator of the Fraction given.

Exa. Reduce $\frac{3}{5}$ *d.* to the Fraction of 1 *L.*

5 Denominator given.
12

60
20

1200 = Denominator required.

So that $\frac{3}{5}$ *d.* is = $\frac{3}{1200}$ *L.* or $\frac{1}{400}$ loweſt.

Exa. 2. Reduce $\frac{4}{7}$ oz. to the Fraction of 1 *C.*

7 Denominator given.
16

112
28

896
224

3136
4

12544 = Denominator required.

So that $\frac{4}{7}$ oz. = $\frac{4}{12544}$ C. = $\frac{1}{3136}$ loweſt.

But it is (perhaps) as convenient to expreſs them in Compound Fractions, and reduce the ſame to Simple ones for the Anſwer. Thus in Exa. 1. $\frac{3}{5}$ *d.* is $\frac{1}{5}$ of $\frac{1}{12}$ of $\frac{1}{20}$ of 1 *L.* = $\frac{3}{1200}$ *L.* In Exa. 2. $\frac{4}{7}$ oz. is $\frac{4}{7}$ of $\frac{1}{16}$ of $\frac{1}{28}$ of $\frac{1}{4}$ of 1 C. = $\frac{4}{12544}$ C.

USE. This Reduction is abſolutely neceſſary before we can add, ſubtract or divide Fractions that belong to different Units.

CASE X.

To expreſs a whole or mixt Number of an inferiour Denomination, by a Fraction of ſome ſuperiour Unit (of the ſame Kind.)

RULE.

RULE. If it is a Simple Number, let it be the Numerator of the Fraction, and for its Denominator, take the Number of that inferiour Species, which makes 1 of the ſuperiour you would have it expreſs'd in, and you have the Fraction required.

Exa. 1. Expreſs 4 *d.* in the Fraction of 1 *L.* In 1 *L.* are 240 *d.* wherefore 4 *d.* is $\frac{4}{240}$, or $\frac{1}{60}$ *L.* loweſt.

Exa. 2. Expreſs 3 qrs. in the Fraction of 1 *ſh.* In 1 *ſh.* are 48 *far.* wherefore 3 *far.* is $\frac{3}{48}$ *ſh.* or $\frac{1}{16}$ *ſh.* loweſt.

Exa. 3. Expreſs 2 lib. in the Fraction of 1 C. In 1 C. are 112 lib. wherefore 2 lib. is $\frac{2}{112}$ C. or $\frac{1}{56}$ loweſt.

2. If it is a mixt Number, let it be firſt reduced to a ſimple Number of the loweſt Species mentioned, and that Number is the Numerator; and for the Denominator, take the Number of that Species you reduce your mixt Number to, that makes 1 of the ſuperiour. *Exa.* 1. Expreſs 3 *ſh.* 8 *d.* in the Fraction of 1 *L.* 3 *ſh.* 8 *d.* is 44 *d.* wherefore there being 240 *d.* in 1 *L.* 3 *ſh.* 8 *d.* is $\frac{44}{240}$ *L.* or $\frac{11}{60}$ loweſt.

Exa. 2. Expreſs 6 *ſh.* 2 *d.* 3 *far.* in the Fraction of 1 *L.* The mixt Number reduced to a ſimple one makes 299 *f.* for the Numerator, and the Denominator is 960, there being that Number of *far.* in 1 *L.* ſo the Anſwer is $\frac{299}{960}$ *L.*

Exa. 3. Expreſs 4 lib. 7 oz. in the Fraction of 1 C. 4 lib. 7 oz. is 71 oz. then becauſe in a C. are 1792 oz. the Anſwer is $\frac{71}{1792}$ C.

USE. This ſeems to be of Uſe principally when a ſimple or mixt Number of an inferiour Species is to be reduced to an equivalent Decimal of an higher.

CASE XI.

To find the Value of a ſimple Fraction in Integers of a lower Species, when the Fraction given is not of the loweſt Denomination.

RULE.

RULE. Multiply the Numerator of the given Fraction by the Number of Units of the next inferiour Species that makes 1 of the Denomination of your Fraction, and divide the Product by its Denominator, the Quote is ſo many Integers of that lower Species, and if there is a Remainder, reduce it to the next inferiour Species, and divide again; and thus go on by reducing and dividing, till you come to the loweſt Species; and the ſeveral Quotes, with the Remainder, if any be, which is always a Fraction of the loweſt Species, are the Anſwer. *Exa.* 1. What is $\frac{2}{3}$ *L.*?

2
20
—
3) 40 (13 *ſh.*
3
—
10
9
—
1 12
—
3) 12 (4 *d.*
12
—

ſh. d.
Anſr. 13: 4.

Exa. 2. What is the Value of $\frac{4}{5}$ *ſh.*?

4
12
—
5) 48 (9 *ſh.*
45
—
3
4
—
5) 12 (2 *d.*
10
—
2

d. f.
Anſr. 9: $2\frac{2}{5}$

Exa.

Exa. 3. What is the Value of $\frac{136}{356}$ C.?

34
4
356) 136 (0 qrs.
28
1088
272
356) 3808 (10 lib.
356
248
16
1488
248
356) 3968 (11 oz.
356
408
356
52
16
356) 832 (2 dr.
712
120

lib. oz. dr.
Anſr. 10 : 11 : $2\frac{30}{89}$

Exa.

Exa. 4. What is the Value of $\frac{4}{7}$ Boll?

4
16
—
7) 64 (9 pecks.
63
—
1
4
—
7) 4 (0 $\frac{4}{7}$ lip.

p. lip.
Anſr. 9 : 0 $\frac{4}{7}$

USE. The Uſe of ſuch Reduction is evident.

If the given Fraction is Improper, you muſt firſt reduce it, and the Quote is an Integer of the ſame Species with the Fraction; then reduce the Remainder as before. *Exa.* What is the Value of $\frac{77}{8}$ *ſb.*?

8) 77 *ſb.*
9 : 7 : 2

ſb. d. f.
Anſr. 9 : 7 : 2

CHAP. X. *Of Addition of Common Fractions.*

GENERAL RULE. Add together the Numerators for the Numerator of the Sum, and ſetting it over the common Denominator, you have the Anſwer; which, if it is an Improper Fraction, you may reduce it to a whole or mixt Number by Caſe II. Chap. IX. or if it is not in its loweſt Expreſſion, depreſs it by Caſe I.

CASE I.

When the Fractions belong all to the ſame Unit or Integer, and alſo have the ſame Denominator?

I RULE.

RULE. Follow the General Rule, and you have the Answer. *Exa.* What is the Sum of $\frac{1}{5}$ and $\frac{3}{5}$ of any Thing? $\frac{1}{5} + \frac{3}{5} = \frac{4}{5}$ *Exa.* 2. What is the Sum of $\frac{6}{7}$ and $\frac{4}{7}$ of any thing? $\frac{6}{7} + \frac{4}{7} = \frac{10}{7} = 1\frac{3}{7}$ *Answ.* *Exa.* 3. What is the Sum of $\frac{1}{8}$ and $\frac{5}{8}$? $\frac{1}{8} + \frac{5}{8} = \frac{6}{8}$ or $\frac{3}{4}$ *Answ.*

CASE II.

When all the Fractions belong to the same Integer, but have different Denominators.

Reduce them to Fractions having one common Denominator, by Case VII. Chap. IX. and perform the rest of the Work as in the preceding Case. *Exa.* Add $\frac{3}{4}$ to $\frac{5}{6}$. $\frac{3}{4} + \frac{5}{6} = \frac{18}{24}$ and $\frac{20}{24} = \frac{22}{24}$ or $\frac{11}{12}$ Sum. *Exa.* 2. Add together $\frac{3}{8}$, $\frac{2}{3}$ and $\frac{5}{12}$. The Fractions reduced are $\frac{108 + 192 + 120}{288} = \frac{420}{288} = 1\frac{132}{288}$ or $1\frac{11}{24}$ lowest, *Answ.*

CASE III.

When the Fractions have the same or different Denominators, but belong not to the same Integer.

RULE. Reduce them first to Fractions of the same Integer by Case VIII. or IX. Chap IX. and then to one Denominator, if necessary, and apply the general **Rule.** *Exa.* Add $\frac{2}{7}$ *L.* to $\frac{3}{5}$ *ſh.* The Fraction $\frac{3}{5}$ *ſh.* being reduced to the equivalent Fraction of 1 *L.* is $\frac{3}{100}$ *L.* wherefore $\frac{2}{7}$ *L.* $+ \frac{3}{100}$ *L.* when reduced to the same Denominator are $\frac{200 + 21}{700} = \frac{221}{700}$ *L.* for the Sum, $= 6$ *ſh.* 3 *d.* $3\frac{6}{10}$ *f.* or $3\frac{3}{5}$ lowest.

You may either reduce the Fraction of the lower Species to an equivalent of the higher, or contrariwise, for the final Result will be the same in both, as you may try by the preceding Exa. If three or more such Fractions are given, let all the inferior ones be reduced to equivalent Fractions of the highest mentioned, and the rest of the Work is the same as before.

CASE

CASE IV.

To add mixt Numbers.

1. Add the Fractions by themſelves, according to the preceding Caſes, and join their Amount to that of the Integers. Or,

2. Reduce the mixt Numbers to Improper Fractions, and add as above, reducing the Sum to a whole or mixt Number, and the fractional Part (if any be) to its loweſt Terms. *Exa.* Add $2\frac{4}{5}$ to $5\frac{3}{8}$. By the firſt Method I take the two Fractions $\frac{4}{5}$, $\frac{3}{8}$, and reducing them to the ſame Denominator, they become $\frac{32}{40}$ and $\frac{15}{40}$, whoſe Sum is $\frac{47}{40}$ or $1\frac{7}{40}$; to this I add the Sum of the Integers 2 and 5, (*viz.* 7) and the Total Aggregate is $8\frac{7}{40}$ for the Anſwer.

By the ſecond Method I reduce the mixt Numbers to Improper Fractions, and they become $\frac{14}{5}$ and $\frac{43}{8}$, which in the ſame Denominator are $\frac{112}{40}$ and $\frac{215}{40}$, whoſe Sum is $\frac{327}{40} = 8\frac{7}{40}$, as before.

Of theſe two Methods I prefer the firſt, as being eaſieſt and moſt expeditious, eſpecially when the mixt Numbers to be added are three or more.

CASE V.

When all or any are Compound Fractions.

RULE. Reduce them firſt to Simple Fractions, and add according to the preceding Caſes. *Exa.* Add $\frac{1}{3}$ of $\frac{4}{5}$ to $\frac{6}{7}$ of $\frac{3}{8}$ of $\frac{1}{2}$. $\frac{1}{3}$ of $\frac{4}{5} = \frac{4}{15}$, alſo $\frac{6}{7}$ of $\frac{3}{8}$ of $\frac{1}{2} = \frac{18}{112}$ or $\frac{9}{56}$, then $\frac{4}{15} + \frac{9}{56} = \frac{224 + 135}{840}$, whoſe Sum is $\frac{359}{840}$ for the Anſwer.

Take the following Queſtions for your further Exerciſe in this Rule.

1. What is the Sum of $\frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5}$? *Anſr.* $1\frac{17}{60}$.
2. What is the Sum of $\frac{4}{15}$ *L.* $+ \frac{3}{4}$ *ſh.* $+ \frac{2}{3}$ *d.* *Anſr.* $\frac{1087}{1760}$ *L.* $= 6$ *ſh.* 7 *d.* $2\frac{170}{171}$ *far.*

3. What is the Sum of $\frac{3}{11}$ *ſh.* + $\frac{1}{2}$ of $\frac{3}{10}$ *d.* + $\frac{6}{11}$ *far.* *Anſr.* $\frac{43}{140}\frac{77}{60}$ *ſh.* = 3 *d.* $2\frac{41}{935}$ *far.*

4. Add together *L.* $14\frac{3}{8}$ + *L.* $11\frac{1}{9}$ + $19\frac{6}{11}$ *ſh.* + 4 *d.* *Anſr.* *L.* 26: 9: 7: $0\frac{28}{33}$ *far.*

CHAP. XI. *Of Subtraction.*

GENERAL RULE. Subtract the leſſer Numerator from the greater, and the Remainder ſet over the common Denominator gives the Anſwer.

CASE I.

When the Fractions belong to the ſame Unit and have the ſame Denominator, apply the general Rule, and you have the Anſwer.

Exa. 1. From $\frac{3}{5}$ *L.* take $\frac{1}{5}$ *L.* Thus, 3 — 1 = 2 Numerator, therefore $\frac{2}{5}$ *L.* is the Anſwer. *Exa.* 2. From $\frac{19}{3}$ of any thing take $\frac{2}{3}$ of the ſame thing: Thus 19 — 2 = 17, wherefore the Anſwer is $\frac{17}{3}$ = $5\frac{2}{3}$.

CASE II.

When both refer to the ſame Integer, but have different Denominators.

RULE. Reduce them to the ſame Denominator, and then apply the Rule. *Exa.* 1. From $\frac{3}{4}$ take $\frac{1}{6}$. The Fractions reduced to the ſame Denominator, are $\frac{18}{24}$ and $\frac{4}{24}$; the Difference of their Numerators is 14, which ſet over the common Denominator, gives $\frac{14}{24}$ or $\frac{7}{12}$ for the Anſwer. *Exa.* 2. From $\frac{5}{6}$ take $\frac{2}{3}$. Thus $\frac{5}{6} - \frac{2}{3} = \frac{15 - 12}{18} = \frac{3}{18}$ or $\frac{1}{6}$ loweſt, for the Difference.

CASE III.

When the Fractions have the ſame or different Denominators, but belong not to the ſame Integer.

RULE. Reduce them firſt to Fractions of the ſame Integer, next to Fractions having the ſame Denominator, (if need be) and apply the general Rule. *Exa.* From $\frac{2}{7}$ *L.* take $\frac{3}{5}$ *ſh.* $\frac{3}{5}$ *ſh.* is $\frac{3}{100}$ *L.* wherefore $\frac{2}{7}$ *L.* $- \frac{3}{100}$ *L.* $= \frac{200 - 21}{700} = \frac{179}{700}$ *L.* $=$ 5 *ſh.* 1 *d.* $1\frac{17}{35}$ *Anſwer.*

If you want the Difference betwixt any two ſuch Fractions in Integers of known Species, you need not reduce them after the above Method, but reduce them at firſt to their known Values, and then ſubtract. So in the laſt Example, $\frac{3}{7}$ *L.* is (by laſt Caſe of Reduction) found to be 5 *ſh.* 8 *d.* $2\frac{2}{7}$ *far.* and $\frac{3}{5}$ *ſh.* is, by the ſame Caſe, 7 *d.* $0\frac{4}{5}$ *far.* which ſet down, and ſubtract, as in the Margin.

s.	*d.*	*f.*
5 :	8 :	$2\frac{2}{7}$
:	7 :	$0\frac{4}{5}$
5 :	1 :	$1\frac{17}{35}$

CASE IV.

When one or both are mixt Numbers.

1. If one or both are mixed, and the Fractions have the ſame Denominator, there is no need of any Reduction, but the Work may be performed as in the following Examples.

Exa. 1.	*Exa.* 2.	*Exa.* 3.	*Exa.* 4.	*Exa.* 5.
From $6\frac{7}{8}$	6	$6\frac{7}{8}$	$6\frac{7}{8}$	7
Take 4	$4\frac{7}{8}$	$4\frac{7}{8}$	$4\frac{7}{8}$	$1\frac{4}{15}$
Rem. $2\frac{7}{8}$	$1\frac{1}{8}$	$2\frac{4}{8}$ or $\frac{1}{2}$	$1\frac{4}{8}$ or $\frac{1}{2}$	$6\frac{11}{15}$

2. But if they are both mixt, and the Fractions have different Denominators, you muſt reduce the mixt Numbers into Improper Fractions, and theſe to the ſame Denominator, and apply the general Rule. *Exa.* From $6\frac{1}{4}$ take $4\frac{3}{7}$. When reduced they become $\frac{27}{4}$ and $\frac{31}{7}$; then in the ſame Denominator, they are $\frac{189}{28}$ and $\frac{124}{28}$, ſo that their Difference is $\frac{65}{28}$ or $2\frac{9}{28}$. Or you may reduce the 2 Fractions to the ſame Denominator, and annexing them to their proper Inte-

gers, ſubtract as in Part I. of this Caſe. Thus in the Exa. $6\frac{3}{4} - 4\frac{3}{7}$, $\frac{3}{4}$ and $\frac{3}{7}$ in the ſame Denominator, are $\frac{21}{28}$ and $\frac{12}{28}$, which I annex to their proper Integers, and ſubtract, as in the Margin, beginning with the Fractions.

$$\begin{array}{r} 6\frac{21}{28} \\ 4\frac{12}{28} \\ \hline 2\frac{9}{28}. \end{array}$$

CASE V.

When one or both are Compound Fractions:

RULE. Reduce them to Simple ones, and to one Denominator, and then apply the general Rule.

Exa. From $\frac{4}{7}$ of $\frac{3}{8}$ take $\frac{3}{5}$ of $\frac{2}{9}$. Thus $\frac{4}{7}$ of $\frac{3}{8} - \frac{3}{5}$ of $\frac{2}{9} = \frac{12}{56} - \frac{6}{45}$, or in their loweſt Terms $\frac{3}{14} - \frac{2}{15}$ $= \frac{45-28}{210}$, ſo that their Difference is $\frac{17}{210}$. And ſo of others.

More Examples for further Exerciſe in this Rule may be ſuch as follow.

1. What is the Difference betwixt $\frac{1}{1000}$ and $\frac{1}{10000}$. *Anſr.* $\frac{9}{10000}$.

2. What is the Difference betwixt $\frac{8}{15}$ *ſh.* and $\frac{1}{2}$ of $\frac{4}{5}$ *far.* *Anſr.* 6 *d.* $1\frac{11}{15}$ *far.*

3. From $20\frac{4}{5}$ *L.* $13\frac{6}{7}$ *ſh.* $1\frac{1}{2}$ *d.* take 19 $\frac{8}{15}$ *L.* 18 $\frac{9}{11}$ *ſh.* $10\frac{3}{5}$ *d.* $2\frac{3}{8}$ *far.* *Anſr.* $\frac{4}{15}$ *L.* $14\frac{3}{77}$ *ſh.* $1\frac{2}{10}$ *d.* $1\frac{5}{8}$ *far.* = 19 *ſh.* 6 *d.* $3\frac{2}{30}\frac{93}{80}$ *far.*

4. From 1 C. take $\frac{1}{3}$ qrs. 19 $\frac{1}{8}$ lib. $4\frac{2}{3}$ oz. 10 dr. *Anſr.* $2\frac{2}{3}$ qrs. $7\frac{7}{8}$ lib. $10\frac{1}{3}$ oz. 6 dr. = 2 qrs. 27 lib. 3 oz. $10\frac{4}{15}$ dr.

5. *A* has $\frac{7}{10}$ of a Ship, and *B* has $\frac{3}{4}$ of the ſame, what is the Difference of their Shares? *Anſr.* $\frac{1}{20}$.

CHAP.

CHAP. XI. *Multiplication in Common Fractions.*

THE multiplying any Number, whether Integral or Fractional by a Fraction, is the taking ſuch a Part or Parts of that Integer or Fraction, as the multiplying Fraction expreſſeth.

GENERAL RULE. Multiply the two Numerators the one by the other, as alſo the two Denominators into one another, for the Numerator and Denominator of the Product reſpectively, which, if it happen to be in high Terms, let it be depreſs'd to its loweſt; or, if it is an Improper Fraction, reduce it to its equivalent whole or mixt Number. You may alſo abbreviate the Fractions to be multiplied (when poſſible) before you begin, and ſo they become more manageable.

CASE I.

When they are both Simple Fractions referred to the ſame Integers.

RULE. Obſerve the general Rule. *Exa.* Multiply $\frac{3}{4}$ by $\frac{2}{3}$. Thus, $\frac{3}{4} \times \frac{2}{3} = \frac{6}{12}$ or $\frac{1}{2}$, Product. *Exa.* 2. $\frac{5}{8} \times \frac{3}{7} = \frac{15}{56}$ *Anſr.* *Exa.* 3. $\frac{14}{6} \times \frac{1}{8} = \frac{14}{48}$ or $\frac{7}{24}$ Product.

CASE II.

When the one is a whole Number and the other a Fraction.

RULE. Reduce the whole Number to a Fraction, and then apply the general Rule. *Exa.* Multiply 6 by $\frac{2}{9}$. Thus $\frac{6}{1} \times \frac{2}{9} = \frac{12}{9} = 1\frac{3}{9}$ loweſt, *Anſr.* *Exa.* 2. Multiply 56 by $\frac{4}{7}$. Thus $\frac{56}{1} \times \frac{4}{7} = \frac{224}{7} = 32$ *Anſr.* Or multiply the whole Number by the Numerator, and divide the Product by the Denominator,

CASE

CASE III.

When one or both are mixt Numbers.

RULE. Reduce the mixt Numbers to Simple Fractions, and multiply as before. *Exa.* Multiply 16 by $2\frac{1}{2}$. Thus $\frac{16}{1} \times \frac{5}{2} = \frac{80}{2} = 40$ *Anſr.* *Exa.* 2. $3\frac{1}{4} \times 8 = \frac{13}{4} \times \frac{8}{1} = \frac{104}{4} = 26$ *Anſr.* *Exa.* 3. Multiply $5\frac{1}{6}$ by $13\frac{2}{7}$. Thus $\frac{31}{6} \times \frac{93}{7} = \frac{2883}{42} = 68\frac{9}{14}$ *Anſr.*

1. But if the Multiplier is a whole Number, and the Multiplicand mixt (or you may make either of them Multiplier, ſeeing the Product will be the ſame) you need not reduce, but beginning with the Fraction, multiply it, (*viz.* its Numerator) and if the Product is Improper, reduce it, ſetting down the Remainder, and carrying the Integral Part or Quote of the Diviſion, to the next Product. *Exa.* Multiply $8\frac{3}{4}$ by 6. Here I ſay 6 times $\frac{3}{4}$ is $\frac{18}{4}$, which is 4 to carry, and $\frac{2}{4}$ or $\frac{1}{2}$ to be ſet down; then 6 Times 8 is 48 + 4 carried is 52. See the Margin.

$8\frac{3}{4}$
6
$52\frac{2}{4}$ or $\frac{1}{2}$

Exa. 2. Multiply 5 by $19\frac{2}{3}$. Here I take the Integral Number for the Multiplier, becauſe the Work will be eaſier.

$19\frac{2}{3}$
5
$98\frac{1}{3}$

Alſo if one of them is the Product of 2 or more Digits, the Anſwer may be found without Reduction as before. *Exa.* 1. Multiply $17\frac{5}{8}$ by 18. Here I multiply by 3 and 6 inſtead of 18. See the Margin.

$17\frac{5}{8}$
3
$52\frac{7}{8}$
6
$317\frac{3}{8}$ or $\frac{1}{4}$

Exa 2. Multiply $135\frac{6}{7}$ by 112. Here I multiply by 4, 4, 7, becauſe $4 \times 4 \times 7 = 112$, and the Anſwer is 15216, as you may try at your Leiſure.

2. Alſo when both are mixt, the Work may ſometimes be performed without Reduction too, and ſo much

much compendized, as in the following Example to multiply $48\frac{2}{3}$ by $15\frac{3}{4}$.

$48\frac{2}{3}$
$15\frac{3}{4}$
240
48
36
$10\frac{6}{12}$ or $\frac{1}{2}$
$766\frac{1}{2}$ *Anſr.*

Here I multiply 48 by 15, ſetting down the partial Products as in common Multiplication, then I multiply 48 by the Fraction $\frac{3}{4}$, and the Product 36 I place under the other two Products, *viz.* in Units and 10's Place; next I multiply 15 by the other Fraction $\frac{2}{3}$, and the Product 10 I ſet under the reſt, according to the laſt Form; laſtly, I multiply the two Fractions together, and their Product $\frac{6}{12}$ or $\frac{1}{2}$ I annex to the Integers, and then add all together. But this Method can be conveniently uſed only when the Integers are diviſible by the Denominators of the alternate Fractions without a Remainder, that is, when theſe Denominators are aliquot Parts of their oppoſite Integers.

CASE IV.

When one or both are Compound Fractions.

RULE. Reduce to Simple ones, and proceed as before. *Exa.* Multiply $\frac{2}{3}$ of $\frac{4}{5}$ by $\frac{3}{6}$ of $\frac{1}{4}$; when reduced they become $\frac{8}{15} \times \frac{1}{8} = \frac{8}{120}$ or $\frac{1}{15}$ *Anſr.*

1. When the Multiplicand is a mixt applicate Number, with a Fraction annexed to its loweſt Species, and the Multiplier a Simple Fraction or reduced to ſuch, then reduce the mixt Number to its loweſt Species mentioned, next to a Fraction of the Denominator of the Fraction annex'd, and then multiply according to the general Rule, the Product is the Anſwer in that loweſt Species, which you may reduce as you ſee neceſſary. *Exa.* Multiply *L.* 10: 8: $4\frac{1}{5}$ by $2\frac{3}{8}$; being reduced they become $2500\frac{1}{5}$ *d.* $\times \frac{19}{8} = \frac{12501}{5}$ *d.* $\times \frac{19}{8} = \frac{237519}{40}$ *d.* $= 5937\frac{39}{40}$ *d.* $=$ *L.* 24: 14: $9\frac{39}{40}$ for the Anſwer.

But if the Multiplier is any of the 9 Digits (or if it is the Product of any 2 or more of them) there is no need

need of a previous Reduction of the Multiplicand: Thus to multiply *L.* 38: 8: $4\frac{1}{6}$ by 7, I ſet them down thus, and multiply, carrying at the ſeveral Denominations as in Addition,

L. 38: 8: $4\frac{1}{6}$
7

268: 18: $5\frac{1}{6}$ Product.

Exa. 2. Multiply *L.* 20: 16: $10\frac{3}{8}$ by 24. Here I multiply by 4 and 6, inſtead of 24.

L. 20: 16: $10\frac{3}{8}$
4

83: 7: $5\frac{4}{8}$ or $\frac{1}{2}$ Prod. by 4.
6

Anſr. 500: 4: 9 Product by 6.

3. If the Multiplicand is mixt (as before) and the Multiplier a pure Fraction, and that a ſmall one, multiply the mixt applicate Number by the Numerator of the Fraction, and divide the Product by its Denominator. *Exa.* What is $\frac{3}{4}$ of *L.* 64: 16: 8?

L. 64: 16: 8
3 Numerator.

Denominator 4) 194: 10: 0
48: 12: 6 *Anſwer.*

Or you may firſt divide by the Denominator, and then multiply the Quote by the Numerator. Thus,

4) 64: 16: 8
16: 4: 2
3

48: 12: 6 *Anſr.* as before.

Exa.

Exa. 2. What is $\frac{5}{6}$ of 187 *L.* 13 *ſh.* $4\frac{1}{8}$ *d.*?

187 :	13 :	$4\frac{1}{8}$
		5
6) 938 :	6 :	$8\frac{5}{8}$
156 :	7 :	$9\frac{7}{16}$

By the other Method.

6) 187 :	13 :	$4\frac{1}{8}$
31 :	5 :	$6\frac{3}{48}$
		5
156 :	7 :	$9\frac{7}{16}$

4. When a Fraction, whether Proper or Improper, is to be multiplied by a whole Number, and the Denominator of the Fraction is the ſame with that whole Number, take the Numerator of the Fraction, and it is the Anſwer without any Multiplication. *Exa.* $\frac{7}{8} \times 8 = 7$, ſo $\frac{7}{8} \times \frac{8}{1} = \frac{56}{8} = 7$. *Exa.* 2. $1\frac{2}{5} \times 5 = 12$, for $1\frac{2}{5} \times \frac{5}{1} = \frac{60}{5} = 12$.

5. If you are to multiply a Fraction by 2, you may either multiply the Numerator for the Numerator of the Product, retaining the Denominator given; or you may take $\frac{1}{2}$ of the Denominator, retaining the given Numerator; but this laſt Method is only practicable when the Denominator is even. *Exa.* $\frac{3}{8} \times 2$ is $\frac{6}{8}$ by multiplying the Numerator, or it is $\frac{3}{4}$ by halving the Denominator, which are equivalent.

6. It may ſeem ſtrange, that Multiplication by a Proper Fraction ſhould produce ſomething leſs than the Multiplicand, whereas in whole Numbers the Product is always greater than either Multiplier or Multiplicand, (excepting in one particular Caſe, where the Product is juſt the ſame with either Factor;) but this Difficulty is eaſily removed, by conſidering, that if the multiplying a whole Number by 1 produce the Multiplicand itſelf, to multiply any Number by a Number leſs than 1, (which is a proper Fraction) the Product muſt be leſs than the Multiplicand, in the ſame Proportion as the multiplying Fraction is remov'd from Unity; and this I take to be as clear and intelligible a Reaſon as can be aſſigned for it. *Exa.* To multiply 12 by 1, the Product is preciſely 12, but the Product

duct of 12 multiplied by $\frac{1}{2}$ muſt be only the half of 12, *viz.* 6, becauſe the laſt Multiplier $\frac{1}{2}$ is only the half of the firſt Multiplier 1. So $\frac{3}{4} \times \frac{1}{3}$ gives only $\frac{1}{4}$ for the Product, which is leſs than the Multiplicand or Multiplier. Now you are to obſerve, that this Effect is only produced when one of the Factors is a proper Fraction; for if it is Improper, the whole Number, or the Fraction multiplied, is really increaſed, and conſequently the Product is greater than either the Multiplicand or Multiplier.

Now follow ſome Examples for further Exerciſe in this Rule.

1. If any one Thing coſt $\frac{1}{8}$ *far.* what is the Value of 25 $\frac{1}{3}$ ſuch things at the ſame Rate. *Anſr.* 3 *d.* 3 $\frac{5}{8}$ *far.*

2. I bought 7 $\frac{3}{5}$ Bags of Hops, each weighing 1 C. 2 qrs. 8 $\frac{3}{7}$ lib. how many lib. in all, and what is the total Coſt, at 7 *d.* 1 $\frac{1}{2}$ *far. per* lib.? *Anſr.* 1340 $\frac{6}{7}$ lib. in all, and *L.* 41: 4: 0: 3 $\frac{2}{7}$ total Coſt.

3. What is $\frac{6}{19}$ of *L.* 12: 10: 8: 3 $\frac{1}{5}$? *Anſr. L.* 3: 19: 2: 0 $\frac{16}{95}$.

4. What is the Product of 38 $\frac{2}{3}$ C. 2 $\frac{3}{8}$ qrs. 14 $\frac{1}{12}$ lib. 6 $\frac{4}{9}$ oz. multiply'd by 4 $\frac{4}{5}$? *Anſr.* 189 C. 7 lib. 14 $\frac{44}{135}$ oz.

5. What Number divided by 14 $\frac{1}{8}$, gives 173 $\frac{19}{20}$ for the Quote? *Anſr.* $\frac{407043}{160}$.

CHAP. XII. *Of Diviſion of Common Fractions.*

GENERAL RULE. Multiply the Numerator of the Dividend into the Denominator of the Diviſor,

viſor, and the Product is the Numerator of the Quote; next multiply the Denominator of the Dividend into the Numerator of the Diviſor for the Denominator of the Quote.

CASE I.

When they are both Simple Fractions belonging to the ſame Unit.

RULE. Apply the general Rule, and you have the Anſwer. *Exa.* Divide $\frac{3}{4}$ by $\frac{4}{5}$: Thus $\frac{3}{4} \div \frac{4}{5}$, or thus $\frac{4}{5}) \frac{3}{4} (\frac{15}{16}$ Quote. *Exa.* 2. Divide $\frac{20}{37}$ by $\frac{6}{13}$: Thus $\frac{20}{37} \div \frac{6}{13} = \frac{260}{222} = \frac{130}{111}$ or $1\frac{19}{111}$ loweſt.

CASE II.

When they are both Simple Fractions, but belong not to the ſame Unit.

RULE. Reduce one of them to an equivalent Fraction of the Denomination of the other, and then apply the general Rule. *Exa.* Divide $\frac{2}{7}$ *L.* by $\frac{4}{5}$ *ſh.* Thus $\frac{2}{7}$ *L.* $\div \frac{4}{5}$ *ſh.* is $\frac{2}{7}$ *L.* $\div \frac{4}{100}$ *L.* $= \frac{200}{28} = 7\frac{1}{7}$ loweſt.

CASE III.

When the one is a whole Number, and the other a Fraction.

RULE. Reduce the whole Number to an Improper Fraction, and work as before. *Exa.* $\frac{1}{3}$) 8 is $\frac{1}{3}) \frac{8}{1} (\frac{24}{1}$ or 24 Quote. *Exa.* 2. Divide $\frac{1}{3}$ by 8: Thus $\frac{1}{3} \div \frac{8}{1} = \frac{1}{24}$ Quote. Alſo $40\frac{1}{7}$ divided by 6, quotes $6\frac{29}{42}$.

CASE IV.

When one or both are mixt Numbers.

RULE. Reduce the mixt Numbers to Simple Fractions, and then apply the general Rule. *Exa.* 1. $\frac{4}{11} \div 2\frac{1}{8} = \frac{17}{8}) \frac{4}{11} (\frac{32}{187}$ Quote. *Exa.* 2. $5\frac{3}{4} \div 6\frac{5}{9} = \frac{23}{4} \div \frac{59}{9} = \frac{207}{236}$ Quote.

CASE V.

When one or both are Compound Fractions.

RULE. Reduce the Compound Fractions to Simple ones, and then work by the general Rule.

Exa. $\frac{2}{3}$ of $\frac{1}{2} \div \frac{4}{5}$ of $\frac{1}{7}$, when reduced become $\frac{1}{3} \div \frac{4}{35} = \frac{35}{12}$ or $2\frac{11}{12}$ Quote.

1. You may either write the Dividend and Diviſor thus, $\frac{3}{4} \div \frac{4}{5} = \frac{15}{16}$, where the Dividend ſtands to the left, or you may place them as in Integral Arithmetic thus $\frac{4}{5})\frac{3}{4}(\frac{15}{16}$; only obſerve, that in whatever Form you place them, you make the Dividend always your Standard, by beginning the Multiplication with its Numerator for the Numerator of the Quote, and then multiplying contrariwiſe for the Denominator,,

2. When the Quote is found, you may depreſs it to its loweſt Terms.

3. If they have both the ſame Denominator,, the Quote is ſooneſt found by dividing the one Numerator by the other. *Exa.* $\frac{2}{5})\frac{4}{5}$ (2 Quote. *Exa.* 2. $\frac{12}{17})\frac{4}{17}(\frac{4}{12}$ or $\frac{1}{3}$ Quote.

4. If the Numerator and Denominator,, of the Dividend can be divided without a Remainder by the Numerator and Denominator,, of the Diviſor reſpectively,, divide; and the two Quotes are the Anſwer ſought. *Exa.* $\frac{2}{3})\frac{6}{18}(\frac{3}{6}$ or $\frac{1}{2}$ loweſt. *Exa.* 2. $\frac{3}{7})\frac{9}{28}(\frac{3}{4}$ Quote. Or if you can find any Number which will divide the Terms of the Diviſor or Dividend without a Remainder, divide, and take theſe Numbers inſtead of the given ones, and ſo the Quote will come out in lower Terms than if no ſuch Diviſion had been made. *Exa.* $\frac{3}{12})\frac{8}{17} = \frac{1}{4})\frac{8}{17}(\frac{32}{17} = 1\frac{15}{17}$ Quote. *Exa.* 2. $\frac{7}{21})\frac{5}{20} = \frac{1}{3})\frac{1}{4}(\frac{3}{4}$ Quote. And ſo of others.

5. When the Numerator of the Dividend can be divided without a Remainder by the Numerator of the Diviſor, divide, and by the Quote multiply the Denominator,, of the Diviſor for the Numerator of the Anſwer, retaining the Denominator,, of the Dividend for the Denominator,, of the Quote. *Exa.* $\frac{2}{5})\frac{4}{11}(\frac{10}{11}$ Quote. *Exa.* 2. $\frac{3}{8})\frac{12}{17}(\frac{32}{17}$ or $1\frac{15}{17}$ Quote.

6. When the Denominator,, of the Dividend is diviſible without a Remainder by the Denominator,, of the Diviſor,

Diviſor, divide, and multiply the Quote by the Numerator of the Diviſor for the Denominator of the Anſwer, retaining the Numerator of the Dividend for the Numerator. *Exa.* $\frac{4}{7}) \frac{5}{21} (\frac{5}{12}$. *Exa.* 2. $\frac{3}{8}) \frac{7}{24} (\frac{7}{9}$ Quote.

7. When the Numerator of the Dividend can be divided without a Remainder by the Numerator of the Diviſor, and alſo the Denominator of the Diviſor by the Denominator of the Dividend, then divide the one Numerator by the other, as alſo the one Denominator by the other, and the Product of theſe two Quotes is the Anſwer ſought. *Exa.* $\frac{2}{15}) \frac{4}{5} (6$ Quote. *Exa.* 2. $\frac{5}{27}) \frac{15}{9} (9$ Quote.

8. When the Diviſor is an abſtract Fraction, and the Dividend a ſimple applicate Number, work by the general Rule, and the Quote is applicate to the ſame Integer with the Dividend. *Exa.* $\frac{6}{7}) \frac{4}{5}$ *L.* or of what Sum of Money is $\frac{4}{5}$ *L.* the $\frac{6}{7}$ Part: Thus $\frac{6}{7}) \frac{4}{5} (\frac{14}{15}$ *L.* $=$ 18 *ſh.* 8 *d.* ſo that $\frac{4}{5}$ *L.* is $\frac{6}{7}$ of $\frac{14}{15}$ *L.* or of 18 *ſh.* 8 *d.* But if both are applicate, the Quote is abſtract. *Exa.* $\frac{6}{7}$ *L.*) $\frac{4}{5}$ *L.* $(\frac{14}{15}$; which denotes that $\frac{6}{7}$ *L.* is contained in $\frac{4}{5}$ *L.* $\frac{14}{15}$ times, which is very near once, and conſequently $\frac{6}{7}$ *L.* is nearly equal to $\frac{4}{5}$ *L.* that is $\frac{6}{7}$ *L.* exceeds $\frac{4}{5}$ *L.* by $\frac{2}{35}$ *L.* Another *Exa.* What Part or Parts of 2 C. 3 qrs. $12\frac{1}{5}$ lib. is 6 C. $2\frac{1}{2}$ qrs.? When reduced they become $\frac{242}{1}$ lib.) $\frac{1601}{5}$ lib. $(\frac{1601}{3710}$ *Anſr.*

9. When the Diviſor is an abſtract mixt Number, and the Dividend a mixt applicate, reduce the Diviſor to an Improper Fraction, and the Dividend to the loweſt Species mentioned, and then apply the general Rule. *Exa.* $3\frac{2}{3}$) 16 *L.* 10 *ſh.* 4 *d.* (by Reduction $\frac{11}{3}$) $\frac{3964}{1}$ *d.* $= \frac{11892}{11}$ *d.* $=$ 1081 $\frac{1}{11}$ *d.* $=$ *L.* 4: 10: $1\frac{1}{11}$ *Anſwer.* But this is ſooner done by dividing the mixt applicate Number *L.* 16: 10: 4 by the Numerator of the Improper Fraction $\frac{11}{3}$, and then multiplying the Quote by its Denominator. Thus,

11) 16: 10: 4
1: 10: $0\frac{4}{11}$
3 Denominator.

4: 10: $1\frac{1}{11}$ *Anſr.* as before.

10. A Fraction is halved or divided by 2, by halving its Numerator if even, or by doubling the Denominator, if the Numerator is odd. Thus $\frac{4}{7} \div 2$ is $\frac{2}{7}$, and $\frac{5}{8} \div 2$ is $\frac{5}{16}$.

11. As in Multiplication, if the Multiplier is a Proper Fraction, the Product is leſs than the Multiplicand; ſo in Diviſion, if the Diviſor is a Proper Fraction, the Quote is greater than the Dividend: Both which are contrary to Multiplication and Diviſion in whole Numbers. I have already briefly accounted for the former, and now it remains that I explain the latter, which I ſhall do after this manner. Diviſion finds how oft one Number is contained in another, conſequently the Quote declares, how oft the Diviſor is contained in the Dividend. Now to divide by a Proper Fraction is only finding how oft that Proper Fraction is contained in the Dividend, or what Part it is of the Dividend. Hence it is plain, that if the Dividend is a whole Number, the Quote muſt be greateſt; becauſe if when the Diviſor is 1, the Quote is preciſely the ſame with the Dividend, it follows, that when the Diviſor is leſs than 1, the Quote muſt be greater than the Dividend, (for the greater the Diviſor the leſs is the Quote, and *vice verſa*;) ſo if 24 is to be divided by $\frac{1}{8}$, it is evident that $\frac{1}{8}$ Part is contained in 24 more than 24 times; for 1 is contained in 24, 24 times, therefore $\frac{1}{8}$, which is leſs than 1, muſt be contained oftner, and by conſequence the Quote muſt be greateſt. Moreover, tho' the Dividend is a Proper Fraction leſs than the Diviſor, yet the Quote is a Fraction greater alſo than the Dividend; for Exa. $\frac{2}{3}) \frac{1}{8} (1\frac{1}{8}$. For may not one ask how oft $\frac{2}{3}$ is contain'd in $\frac{1}{8}$, or what Part it is of $\frac{1}{8}$? 'Tis evident $\frac{2}{3}$ cannot be contained

any Integral Number of Times in $\frac{1}{8}$, becauſe the Diviſor here is greater than the Dividend; therefore it muſt be contained ſome Fractional Number of Times, and the Quote $\frac{3}{16}$ denotes that $\frac{2}{3}$ is $\frac{3}{16}$ Parts of the Dividend $\frac{16}{8}$, or that it is contained in $\frac{1}{8}$, $\frac{3}{16}$ Parts of 1; that is, if the Dividend $\frac{1}{8}$ was divided into 16 equal Parts, then $\frac{2}{3}$ is preciſely $\frac{3}{16}$ of theſe Parts.

Or it may be proved thus: The Quote muſt have the ſame Proportion to the Dividend as Unity has to the Diviſor; ſo $\frac{2}{3}$) $\frac{1}{8}$ ($\frac{3}{16}$, the Quote $\frac{3}{16}$: $\frac{1}{8}$: $\frac{1}{1}$: $\frac{2}{3}$. After all, if the Diviſor and Dividend are reduced to the ſame Denominator, the Difficulty is quite removed, for then you divide the one Numerator by the other as in Integers, neglecting the common Denominator, as you may prove at your Leiſure; laſtly, you are to obſerve, that the common Rule for dividing Fractions does not ſo much divide as it produces new Terms, *viz.* a Numerator to be divided by a Denominator; thus $\frac{1}{4}$) $\frac{1}{2}$ ($\frac{4}{2}$, where you ſee the Diviſion produces $\frac{4}{2}$, that is 4 to be divided by 2; but if the Fractions are reduced to one Denominator before you begin, there is no more to do but to make one ſimple Diviſion, and ſo the Quote is found; for Exa. to divide $\frac{1}{2}$ by $\frac{1}{4}$, when reduced they become $\frac{2}{8}$) $\frac{4}{8}$, and ſo the Quote is 2 found by (neglecting the common Denominator 8, and) dividing the one Numerator 4 by the other Numerator 2.

More Examples for Practice.

1. What is the Price of a lib. of any thing, when 12 $\frac{4}{9}$ lib. of the ſame coſt *L.* 5: 17: 10 $\frac{3}{7}$? *Anſr.* 9 *ſh.* 5 *d.* 2 $\frac{125}{196}$ *far.*

2. What Part of *L.* 345: 10: 3: 1 $\frac{3}{8}$ is *L.* 7 $\frac{3}{17}$? *Anſr.* 48 $\frac{1}{9}\frac{36}{36}\frac{21}{96}\frac{9}{00}$.

3. Of what is 11 gall. 3 pints 2 mutchkins 2 $\frac{2}{3}$ gills (*Scots* Meaſure) the $\frac{8}{16}$ Part? *Anſr.* Of 20 gall. 2 p. 3 mut. 3 $\frac{11}{27}$ gills.

4. What Number multiplied by $5\frac{1}{2}$ produces $112\frac{3}{7}$? *Anſr.* $20\frac{3}{11}$.

CHAP. XIII. *Of Decimal Fractions.*

IN Decimal Fractions the Unit, Integer, or whole Thing is divided, or ſuppoſed to be divided into 10 equal Parts, and each of theſe 10 into 10, (ſo that the Unit is hereby divided into 100 Parts) and each of theſe laſt into 10, (whereby the Unit is divided into 1000 Parts, and ſo on infinitely: which Parts are called *Decimal* or Tenth Parts, and any Number of them is called a Decimal Fraction. So that a Common Fraction may have any Number for its Denominator, that is, an Unit there may be divided, or ſuppoſed to be divided into any Number of Parts: But the Denominator of a Decimal is always 10, 100, 1000 or 10000, *&c.* that is, it conſiſts of 1 with any Number of o's annexed; but ſuch Denominators being never expreſs'd, all Operations with them are render'd almoſt as eaſy as with Integers, and there is thereby ſaved a vaſt Trouble, which would be inevitable by uſing the common Expreſſion.

Notation of Decimal Fractions.

RULE. Firſt ſet down the Numerator of the Fraction, whether Proper or Improper, then conſider how many o's there are in the Denominator, and beginning at the Place of Units, or right hand of the Numerator, reckon towards the left as many Places as the Denominator contains o's; and if there are not ſo many, ſupply the Defect with o's ſet on the left of the ſignificant Figures of the Numerator, and for the laſt

laſt Place of the Denominator (in which always ſtands 1) make a Point thus (.). If the Fraction is Improper, then an equal Number of Figures with that of the o's in the Denominator being pointed off, the Figures remaining towards the left are the Integral Part, and the other the Decimal: All which ſhall be exemplify'd by Fractions tranſlated from the Common to the Decimal Form.

Exa. 1. $\frac{5}{10}$ is expreſs'd in the Decimal Form .5. *Exa.* 2. $\frac{54}{100}$ is .54. *Exa.* 3. $\frac{26}{1000}$ is .026. *Exa.* 4. $\frac{35}{10000}$ is .0035. *Exa.* 5. $7\frac{5}{10}$ is 7.5. *Exa.* 6. $37\frac{84}{100}$ is 37.84. *Exa.* 7. $431\frac{267}{1000}$ is 431.267. *Exa.* 8. $\frac{1}{1000000}$ is .000001.

II. *How to read a Decimal Fraction.*

RULE. Take the whole Rank of Figures in the Decimal (neglecting the o's that are prefixed to the left of the ſignificant Figures) for the Numerator, and for the Denominator take 1 with as many o's as the Decimal contains Figures both ſignificant and Cyphers. *Exa.* ,5 is read $\frac{5}{10}$; ,026 is $\frac{26}{1000}$; ,0035 is $\frac{35}{10000}$; ,000001 is $\frac{1}{1000000}$, *&c.* So that you ſee they are hereby reduced back again from the Decimal to the common Form. But there are ſeveral other Ways of reading or expreſſing (in Words) a Decimal Fraction; as,

1. By calling the Figure next to the Point ſo many Primes, the ſecond Figure ſo many Seconds, the third Figure ſo many Thirds, *&c.* Thus the Decimal .5873 read after this Way is 5 Primes, 8 Seconds, 7 Thirds and 3 Fourths.

2. Others would expreſs the ſame Decimal thus, 5 thouſand 8 hund. ſeventy-three Fourths.

3. The other Method, as practiſed by ſome, is nothing elſe than calling it ſo much of a Decimal as the Numerator expreſſes; ſo the laſt propoſed Decimal read this Way is 5 thouſ. 8 hundred ſeventy-three of

a Decimal, or it is five, eight, ſeven, three of a Decimal; which, tho' not the moſt natural, I think the ſhorteſt and eaſieſt Method of expreſſing them, and as intelligible as any other, eſpecially, if their Nature and Geneſis be well underſtood.

And now for further Explication of what I have already ſaid, the Learner is carefully to conſider the following Remarks.

1. A Decimal Fraction is ſo many 10ths of the whole, or it is ſo many 10ths of a 10th, or ſo many 10ths of $\frac{1}{10}$ of $\frac{1}{10}$, *&c.* the two laſt of which are Compound Fractions, and being reduced you have their equivalent Simple Expreſſion; for Exa. $\frac{3}{10}$ of $\frac{1}{10}$ of $\frac{1}{10}$ is $\frac{3}{1000}$ in the common Form, and .003 in the Decimal. So this Decimal Fraction .6852 (ſuppoſe of 1 *L.*) is 6 tenth Parts, 8 hund. Parts, 5 thouſ. Parts, and 2 ten-thouſ. Parts; or it is 6 tenths, together with 8 tenths of 1 of the laſt tenths, with 5 tenths of 1 of the laſt tenths, with 2 tenths of 1 of the laſt tenths.

2. A Decimal Fraction being written always without its Denominator, is diſtinguiſh'd from a whole Number by a Point, called by ſome the Separatrix, prefixt: but ſome Authors uſe a Comma (,), others a perpendicular Line (|), and others this Mark (L), and ſome uſe other Symbols; but I prefer the Point, as being the ſimpleſt, and alſo becauſe it is moſt common.

3. As the Value of a whole Number is increaſed in a decuple or tenfold Proportion, by annexing a ſignificant Figure or Cypher to the Place of Units or right hand; ſo by prefixing any Figure to the left of a Decimal (and right of the Point) its Value is decreaſed in a ſub-decuple Proportion. Thus 6, which is ſo many Units or ones when ſtanding by itſelf, if you annex to it a ſignificant Figure (ſuppoſe 3) or a Cypher (0), then its Value is changed from 6 to 60, and is read 63 and 60: ſo if 6 is a Decimal it is .6

or

or $\frac{6}{10}$, but if you prefix 3 to it, then it is only $\frac{6}{100}$ (and with the 3 prefixt makes $\frac{36}{100}$) its Value being changed from .6 to .06 or $\frac{6}{100}$. If instead of 3 you prefix 0, it is precisely $\frac{6}{100}$; if another 0 is prefixt, it is only $\frac{6}{1000}$ or .006; and if you prefix another 0, it it becomes $\frac{6}{10000}$ or .0006; and by still prefixing 0's or significant Figures to its left, its Value is decreased infinitely.

4. As Cyphers set on the left of Integers neither increase nor diminish their Value, so Cyphers set on the right of Decimals neither increase nor diminish their Value. Thus if you should prefix ever so many 0's to 6 being an Integer, for Exa. 0006, it is only 6. In like manner, supposing 6 to be a Decimal (thus .6) if you annex to its right any Number of 0's, it will be but .6 or $\frac{6}{10}$, because whatever Number of 0's you annex, the same Number is supposed to be annexed to its Denominator, and so by cutting off an equal Number of 0's from Numerator and Denominator, (as was taught in Case I. *of Reduct. of Com. Fract.*) it is reduced to its first State.

5. A Proper Fraction in Decimals hath all its Figures standing on the right of the Point, because the Denominator must have more Places than the Numerator, and therefore the Point must fall without the Figures of the Numerator; but an Improper Decimal Fraction has Figures on both sides of the Point, *viz.* the Integral Part on its left, and the Fractional on the right; thus $\frac{37}{100}$ is .37, but $\frac{378}{100}$ is 3.78.

6. A Proper Decimal Fraction may be divided into as many lesser ones as it contains significant Figures, their Numerators being the several Figures of the Numerator given, and their Denominators having as many 0's as there are Places after the Point to these severally. Thus .536 is .5 + .03 + .006; also .0481 is .04 + .008 + .0001.

CHAP.

CHAP. XIV. *Of Addition of Decimal Fractions.*

ADdition of Decimals is the ſame with that of whole Numbers, reſpect being had to the true and orderly placing of the Numbers to be added; for which obſerve the following

RULE. Whether they are pure Decimals or mixt (that is, Integers and Decimals) diſpoſe them ſo the one under the other, that all the Points may ſtand in one Column, and the Figures in ſeveral Columns, each according to its Place and Degree; then beginning at the right hand, add together the Figures in each Column, and for every 10 in the Sum carry 1 to the next Column (as in Addition of ſimple abſtract Numbers in Integral Arithmetic), ſetting down the Exceſs, and minding to place the Point in the Sum under thoſe in the Numbers given.

Exa. 1.	*Exa.* 2.	*Exa.* 3.
yds.	L.	
.17	.5	.56
.05	.125	.908
.13	.05	.137
.258	.8	.02
.312	.25	.384
.076	.025	.605
Sum .996	Sum 1.75	Sum 2.614

Exa. 4.	*Exa.* 5.	*Exa.* 6.
137.354	48.057	24.4183
.27	3.16	5.0056
565	5.	573.673
15.138	18.73	.3527
.6	.62	46.975
.87904	.033	.1824
210.74104	75.6	650.607

These

Theſe Examples need (I ſuppoſe) no Illuſtration, and therefore I ſhall finiſh this Rule, after I have obſerved, 1. That if the Sum of the Decimals in the firſt Column, *viz.* to the right hand, is a preciſe Number of 10's, the 0 needs not be ſet down, but you may proceed, carrying your Number of 10's to the next Column; and you may do the ſame, if there ſhall happen a 0 in the next place. (See Exa. 2, 5, 6, where the 0's Places are left vacant.) But after a ſignificant Figure theſe Cyphers muſt not be neglected, See Exa. 4. and 6.

CHAP. XV. *Subtraction in Decimal Fractions.*

RULE. Set down the Minuend and Subtrahend the ſame Way as was directed in Addition; *viz.* by placing the two Points the one under the other, and each Figure of the Subtrahend orderly under thoſe of the Minuend, and begin at the right hand and ſubtract as in Integers, paying 1 to the next Place, when you have Occaſion to borrow 10.

	Exa. 1.	*Exa.* 2.	*Exa.* 3.	*Exa.* 4.
Minuend	.738	.405	.0083	.27348
Subtrahend	.462	.032	.006	.198
	.276	.373	.0023	.07548

1. If one or both is a mixt Number, ſet down the Integral Part towards the left hand, and the Fractional on the right, obſerving the above Direction.

	Exa. 5.	*Exa.* 6.	*Exa.* 7.
From	16.2805	143.81	60.437
Take	.135	7.9	59.362
	16.1455	135.91	1.075

2.

2. If the Decimals in the Minuend are fewer than those in the Subtrahend, then as many o's must be placed, or supposed to be placed over the Subtrahend's Figures as will fill up the Defect.

Exa. 8.	*Exa.* 9.	*Exa.* 10.	*Exa.* 11.
58.2	12.75	1.	100.
7.25	.6831	.4593	99.999
50.95	12.0669	.5407	.001

3. As in Addition, so likewise here, if a Cypher shall happen on the right of the first significant Figure of the Remainder, the same may be neglected, as being of no Use.

Exa. 12.	*Exa.* 13.	*Exa.* 14.
.485	14.137	20.0084
.165	5.237	.7084
.32	8.9	19.3

CHAP. XVI. *Multiplication in Decimal Fractions.*

GENERAL RULE. Set down the Multiplicand and Multiplier as in whole Numbers, and multiply them together as such; then point off so many Figures to the right of the Product as there are Decimal Places in both Factors, and if there happen not to be so many in the Product, prefix as many o's on the left of the Decimals found, as is the Difference; which Product, according to the Quality of the Factors, will be either a pure Fraction, (as in Exa. 1, 2, 3.) or a mixt Number, (as in Exa. 4, 5.) or an Integer (as in Exa. 6.)

Exa.

Exa. 1.	*Exa.* 2.	*Exa.* 3.
Mult. .63	.046	4
by .45	.005	.23
315	.00023	.92
252		
.2835		

Exa. 4.	*Exa.* 5.	*Exa.* 6.
.372	26.185	6.25
68	3.54	6.4
2976	10474	2500
2232	130925	3750
25.296	78555	40.
	92.6949	

1. When a Decimal Fraction or a mixt Number is to be multiplied by 10, 100, or 1000, *&c.* there is nothing to do but to remove the Point as many Places towards the right hand as there are o's in the Multiplier, (such o or o's on the right of the Product being to be neglected.) Thus,

If the Decimal Fraction .5736 was to be multiplied by

10, 100, 1000, 10000 } the Product would be { 5.736, 57.36, 573.6, 5736.

For .5736 × 100 = 57.3600, where the two o's are insignificant.

In like manner, if the mixt Number 38.754 was to be multiplied by

10, 100, 1000, 10000, *&c.* } the Product would be { 387.54, 3875.4, 38754., 387540. *&c.*

2. When a Decimal Fraction is to be multiplied by .1, .01, .001, .0001, there is no more to be done, but to prefix to the Multiplicand as many o's (with the Decimal Point on the left of all) as there are Figures in the Multiplier. So

.5736	multiplied by	.1	produces	.05736
		.01		.005736
		.001		.0005736
		&c. .0001		.00005736, &c.

3. Becauſe the Work of Multiplication of Decimals, when there are many Places in the Multiplicand or Multiplier, or both, is very tedious; and ſeveral of the Decimal Places of the Product when found, being to be rejected as of no uſe, ſome Authors propoſe a compendious Method of performing the Work true, or nearly ſo, to as many Decimal Places in the Product as you incline, *viz.* to 2, 3, 4, *&c.* Places (after the Point) as you ſhall think you may have uſe for; for which I ſhall ſet down the Rule, and make an Obſervation or two upon the Advantage that is gain'd by this Method.

RULE. Having written down the Multiplicand, conſider how many Decimal Places you incline to have in the Product, and ſet the Units Place (*viz.* of Integers) of the Multiplier under that Decimal Place of the Multiplicand which ſtands as far from the Point as you deſign the Product ſhould have Decimal Places, that is, under the firſt Figure after the Point, if you would have only one Decimal Place in the Product, under the ſecond if you would have 2, *&c.* then ſet the remaining Figures of the Multiplier in the reverſe Order, and multiply as uſual; only 1*ſt*, you muſt begin with that Figure in the Multiplicand which ſtands over the multiplying Figure, neglecting (as it were) the Figures ſtanding to its right. But, 2*dly*, conſider what would have been carried from the Product of theſe right-hand Figures if you had actually multiplied them, (which may for the moſt part be found by tacitely

citely multiplying the 2 right-hand Figures next that you begin with, and carrying the 10's of the neareſt Figure thereto.) 3*dly*, Set down the partial Products, ſo as they may all ſtand even, or in the ſame Column on the right, which is contrary to the common Method, placing the reſt of the Figures in diſtinct Columns to the left, and then add them together. But yet, 4*thly*, in ſo doing you muſt gueſs as near as you can, what would have been carried from the Sum of the preceding Columns, if none of the Figures in the Multiplicand had been neglected in the ſeveral particular Multiplications, ſo as the ſame may be added to the Sum of the firſt Column. *Exa.* 1. Let it be propoſed to multiply 35.423786 by 3.628, ſo as four Decimals in the Product may be true.

Operation at large.	*Operation* contracted.
35.423786	35.423786
3.628	826.3
283390288	1062713
70847572	212542
212542716	7084
106271358	2833
128.517495608	128.5174

If the Multiplier is a pure Decimal, ſet a 0 in the Units Place (or imagine it ſo) and the Fraction in the reverſe Order, as above. Thus,

Exa. 2. Let 824.6537 bemultiplied by .4657, ſo as there may be three Decimal Places true in the Product.

Operation at large.	*Operation* contracted.
824.6537	824.6537
.4657	7564.0
57725759	329861
41232685	49479
49479222	4123
32986148	577
384.04122809	384.041

Exa.

Exa. 3. Let 59.48576 be multiplied by .0473, so as to have 3 Decimal Places exact in the Product.

Operation at large.	*Operation* contracted.
59.48576	59.48576
.0473	3740.0
17845728	2379
41640032	416
23794304	17
2.813676448	2.813

When there are not so many Decimal Places in the Multiplicand as you would have in the Product, annex as many o's to the Multiplicand as you want; and if it is an Integer, you must reckon the Cyphers added to be Decimals, and therefore a Point is to be set betwixt them and the Integral Part.

And now you may observe (which is very obvious) that after we have multiplied by this contracted Method, it is altogether a Guess what Allowance to make when we come to add, for the Increase of the neglected Figures of the Multiplicand, if they had been actually multiplied, and their several Products set down and added according to common Form; and consequently we may err sometimes in Excess, sometimes in Defect, (for it is impossible to prescribe a Rule which will be exact but in a few Cases,) tho' the Difficulty in making Allowance when multiplying is not so great. Therefore it is not every one that can promise upon (perhaps) tolerable Exactness in using this Method; but only such as have had much Practice therein, and can readily judge what Allowance to make in every common Case.

2. That when we incline to be pretty exact, the best Way is to multiply a Figure or two more than we intend the Product should have of Decimal Places, and after Addition to cast away these as useless.

3.

3. That in ſome Calculations, where much Accuracy is required, this Method ought not to be uſed at all. And even in moſt Caſes, when we uſe the Decimal Way, I would take the common Method of multiplying, I mean by ſetting the Multiplier in the uſual Form; and if the Multiplicand has many Decimal Places, I'd begin to multiply at its third, fourth, *&c.* Place after the Point, as I ſhould judge convenient, rejecting all the Figures to the right of theſe as ſuperfluous, and in the mean time make ſome Allowance for the Increaſe, as nearly true as I could gueſs.

You are further to take notice, that as in Multiplication of Common Fractions, when both Factors are proper Fractions, the Product is leſs than either; ſo in Multiplication of Decimals, when the Factors are both Proper, that is, when they have no Figures on the left of the Point, the ſame Effect is produced, for the Reaſon there aſſigned. *Vide* Exa. 1, 2.

CHAP. XVII. *Diviſion in Decimal Fractions.*

DIviſion of Decimals being reckoned the hardeſt Task in Common Arithmetick, I ſhall endeavour to be very particular in it, and to render it as plain and intelligible as Words will do it, and my deſigned Brevity will permit.

The Difficulty then lies not in the Diviſion itſelf, (for that is the ſame in every reſpect with Integers) but only in qualifying the Quote after the Diviſion is over; that is, in knowing certainly, whether it be all Integral or all Fractional, or a mixt Number; and if the laſt, what the Integral and what the Fractional Part is.

RULE. Conſider the Dividend and Diviſor as whole Numbers, without regarding the Points, and divide in every reſpect as if they were ſuch: But before you begin, if the Diviſor contains a greater Number of Figures than the Dividend, you muſt annex as many o's to the Dividend as will at leaſt make their Number equal, (or you make the Dividend greateſt, which if it is a whole Number, the o's added are to be reckoned its Decimals, and muſt have a Point placed before them;) then proceed in your Diviſion, by adding o's to the Remainders, 'till you have carried the Quote as far as ſhall be neceſſary; and when the Diviſion is finiſh'd, the Quote muſt be qualify'd after this manner, *viz.* Conſider how many Decimal Places there are in the Diviſor, and alſo how many in the Dividend, including in this laſt all the o's you added and made uſe of; then, becauſe the Number of Decimal Places in both muſt either be equal, or the Diviſor has moſt, or the Dividend exceeds the Diviſor, (for there is not a fourth Caſe;) therefore,

1. If the Number of Decimal Places in both are equal, the Quote is a whole Number. *Vide* Exa. 1, 2, 3, 17.

2. If the Diviſor has moſt Decimal Places, annex as many o's as is the Difference, to the right of the Quote, and it is a whole Number. *Vide* Exa. 4, 5, 6, 10, 13, 15, 16.

3. If the Dividend has the greateſt Number of Decimal Places, take the Difference betwixt them and thoſe in the Diviſor, and point off as many to the right of the Quote, ſupplying the Defect (if any be) with a o or o's, and the Quote will either be a Fraction or a mixt Number. *Vide* Exa. 7, 8, 9, 11, 12, 14.

Exa.

Exa. 1.

.167) .835 (5
835

Exa. 2.

.0007) .2996 (428
28
19
14
5
56

Exa. 3.

14.0032) 1036.2368 (74
980224
560128
560128

Exa. 4.

.253) 88.55 (350
759
1265
1265

Exa. 5.

.0872) 17.44 (200
1744

Exa. 6.

.000035) .140 (4000
140

Exa. 7.

.36) .20808 (578
180
280
252
288
288

Exa. 8.

3.8) 100.852 (26.54
76
248
228
205
190
152
152

Exa.

Exa 9.

.06) .804 (.13.4
6
—
20
18
—
24
24
—

Exa. 10.

.0125) .500. (40
500
—

Exa. 11.

.5) .0125 (.025
10
—
25
25
—

Exa. 12.

100) .100 (.001
100
—

Exa. 13.

.1) 100 (1000
1
—
00

Exa. 14.

4) .20 (.05
20
—

Exa. 15.

.2) 4 (20
4

Exa. 16.

.000001) 1 (1000000

Exa. 17.

.05) .10 (2
10
—

The Rule itſelf is ſo very plain, that I ſhall not uſe any Illuſtration for the above Examples, but proceed to lay down another eaſy Method of knowing the true Place and Degree of the Quotient Figures; and it is this:

Conſider under what Figure of the Dividend the Units Place of the Diviſor would ſtand at the firſt Demand, the firſt Figure of the Quote is always of that ſame Place and Value, by which are eaſily known the Places of the other Figures of the Quote.

This will be beſt underſtood by Examples; and to let you ſee the Conſonancy of this Method with the preceding, I ſhall repeat ſome of thoſe already adduced,

ced, and begin with Exa. 1. *viz.* .167) .835 (where becauſe they are pure Fractions, I prefix a 0 to each for their Place of Units, thus 0.167) 0.835 (and find that the Place of Units of the Diviſor ſtands under the Place of Units of the Dividend, as in the Margin; wherefore I conclude according to the Rule, that the Quote 5 is in the firſt Place of Integers alſo.

0.835
0.167

In Exa. 6. the Units Place of the Diviſor ſtands under the fourth Place (or Place of Thouſands of Integers) of the Dividend, conſequently the firſt Figure of the Quote muſt be four thouſand; and becauſe no other Figures follow it, I annex three o's to make it ſo.

0.000035) 0000.140 (
0.000035

The Effect of theſe two Rules is one and the ſame; you may therefore uſe either of them you incline, only you need not ſet down the Diviſor under the Dividend after the above Form, but you may imagine it ſo: I have done it for no other Reaſon than to let you ſee the Method.

1. When a pure or mixt Decimal is to be divided by 10, 100, 1000, or 10000, *&c.* remove the Point as many Places towards the left as there are o's in the Diviſor, and it is done. *Exa.* Let the mixt Number 3875.4 be divided by

10		387.54
100		38.754
1000	and the Quote is	3.8754
10000		.38754
100000		.038754

Which you may prove by actually dividing and qualifying the Quote according to the Rule.

2. When a Decimal Fraction or a mixt Number is to be divided by .1, .01, .001, .0001, *&c.* remove the Decimal Point of the Dividend as many Places towards the right as there are Figures in the Diviſor: Thus if .38754 was divided by

.1

.1
.01
.001
.0001 } the Quote would be { 3.8754
38.754
387.54
3875.4

3. Hitherto I have ſuppoſed no Remainder. I ſhall next ſhew how to value the Quote exactly, when any thing happens to remain after the Diviſion.

RULE. Proceed in your Diviſion as is before taught, 'till all the Figures in the Dividend are taken down; and if the Quote is not then carried to a ſufficient Number of Places, ſet a 0 to the Remainder, and divide, placing another Figure in the Quote; and if this is not ſufficient, add another 0 to the laſt Remainder, and ſo go on by adding 0's to the Remainders, and dividing, 'till your Quote is carried to as many Places as ſhall be judged neceſſary (qualifying the ſame as is already taught,) and value your laſt Remainder thus;

Make it the Numerator of a Fraction, whoſe Denominator is the Denominator of the Dividend, taking in all the Cyphers added and made uſe of in the Operation, and you have the true Remainder; it matters not whether you expreſs it in the Common or Decimal Form, which Remainder if you would add to the Quote already found, to make the ſame complete, you muſt

Take the Remainder in its true Value, as above, and divide it by the Diviſor, taken alſo in its true Value, and the Quote is the additional Part ſought. Or you may do it after this Manner; ſet the Remainder for the Numerator of a Common Fraction, whoſe Denominator is the Diviſor, if the Number of Decimal Places in both is equal. *Vide* Exa. 24. But if the Dividend has moſt, take the Difference and annex as many 0's to the Denominator, *Vide* Exa. 18, 19, 20. Laſtly, if there are more Decimal Places in the Diviſor, take the Difference, and annex as many 0's to the Numerator, and you have the Remainder exact in the

common Expression, to be added to the Quote already found, to make the same complete. *Vide* Exa. 21, 22.

Exa. 18.

.46).942(2.0478 + $\frac{12}{460000}$
92
220
184
360
322
380
368
12

Exa. 19.

.358)487.3(1361.17 + $\frac{114}{3580}$
358
1293
1074
2190
2148
420
358
620
358
2620
2506
14

Exa. 20.

80.279)3876.1375(48.28333 + $\frac{5093}{8027900000}$
321116
664977
642232
227455
160558
668970
642232
267380
240837
265430
240837
245930
240837
5093

In Exa. 18, after I have carried the Quote to 5 Places and qualified it, there remains 12, which is .000012 or in a Vulgar Fraction $\frac{12}{1000000}$, and being to add it to the Quote 2.0478 to make the same complete, I divide it by the Divisor 46 (taken in its true Value, *viz.* .46 or $\frac{46}{100}$, and the Quote is $\frac{1200}{460000}$ for the additional Member. Or I find it thus; I make the Remainder 12 a Numerator to the Divisor 46,

46, thus $\frac{12}{46}$, and becauſe the Decimal Places of the Dividend exceed in Number thoſe of the Diviſor by 4, I annex 4 o's to the Denominator, and it becomes $\frac{12}{460000}$ as before.

Take alſo theſe following Examples.

Exa. 21.

.005)1748.28(349650+$\frac{30}{5}$=349656 preciſely.
15

24
20

48
45

32
30

28
25

3

Exa. 22.

.0006)25384.3(42307000+$\frac{1000}{6}$=42307166$\frac{4}{6}$
24

13
12

18
18

43
42

Rem. 1=$\frac{1000}{6}$

Exa. 23.

.001)4387.6(4387600
4

3
3

8
8

7
7

6
6

0

Exa. 24.

8.37)74896.8(8948+$\frac{204}{837}$
6696

7936
7533

4038
3348

6900
6696

Rem. 204=$\frac{204}{837}$

Ob-

Obſerve 1. When the Number of Decimal Places made uſe of in the Dividend is equal to, or greater than, that in the Diviſor, the Remainder will always be a Proper Fraction, as in Exa. 18, 19, 20; ſee alſo Exa. 24, where the Number of Decimal Places in both being equal, the Remainder with the Diviſor is a Proper Fraction, the reſt of the Quote being Integral.

2 When the Dividend has feweſt Decimal Places, the Remainder will be an Improper Fraction, as in Exa. 21, by which you ſee, that if the Diviſion was carried further on, the Integral Part of the Quote would become greater; ſo that by reducing this Improper Fraction to a whole Number, and adding the ſame to the Quote (already found) in its proper Place, the Quote would have all its Integral Figures, and the Remainder, if any were, would be a Proper Fraction, as in Exa. 22.

Moreover, if the additional Member is an Improper Fraction equal to a whole Number, it ſhews, that after ſo many Steps as the whole Number has Figures, the Diviſion would have been complete without a Remainder, as in Exa. 21.

3. That therefore the firſt part of the Quote may not want 1 of the Truth, and conſequently the Remainder may be a Proper Fraction, the Diviſion is to be continued, 'till the Number of Decimal Places in the Dividend is equal to, or exceed, that in the Diviſor, unleſs the Diviſion is finiſhed without a Remainder before you come to that; for then the Quote qualified according to the Rule will be complete. See Exa. 23.

4. When the Quote is carried to a ſufficient Number of Decimal Places, according to the Nature and Circumſtances of the Queſtion, (and in common Caſes, when the Quote is not to be multiplied, nor very much Accuracy required, five or ſix Places after the Point will be ſufficient; tho' the farther the Diviſion

is continued, the Quote will ſtill be the more exact, and want leſs of its true Value) the Remainder may be ſafely neglected as of no uſe. But how far the Diviſion is to proceed muſt be left to every one's own Diſcretion, ſince no Rule can be preſcribed for that Purpoſe. However, you'll find, when we come to Reduction of Decimals, how to manage the Quote, ſo as it ſhall not want an Unit of the loweſt Denomination, nor any aſſignable Fraction of its true Value.

CHAP. XVIII. *Reduction of Decimals.*

REduction of Decimal Fractions we ſhall conſider under the following Caſes, *viz.*

1. To reduce a Common (Proper) Fraction to a Decimal one equivalent, or nearly ſo.
2. To reduce a Decimal of an higher Species to an equivalent one of a lower.
3. To reduce a Decimal of an inferior Species to an equivalent one of an higher.
4. To reduce Integers of a lower Species or Denomination, to equivalent Decimals of higher Species.
5. To find the Value of a Decimal Fraction in Integers of known Species.
6. To reduce a Decimal Fraction to a Common one.

CASE I.

To reduce a Common Fraction to a Decimal.

RULE. Say, by the Rule of Three: Numbers, as the Denominator of the Common Fraction is to its Numerator, ſo is 10, 100, 1000, *&c.* (as you intend the Decimal ſhould have Places) to the Anſwer.

Or rather thus:

Add a competent Number of o's to the Numerator of the Common Fraction (*viz.* 'till it be equal to, or

or greater than its Denominator,) and dividing by the Denominator, the Quote is the Decimal ſought. Obſerve, that if the Diviſion is not finiſhed, after you have made uſe of all the o's you added to the Numerator, you may annex more to the Remainders (as was hinted at already) and continue your Diviſion as far as you pleaſe, qualifying the Quote according to the Rule. Alſo ſet a Point betwixt the o's you added to the Numerator and its own proper Figures, that you may not miſtake when qualifying the Quote, which you do (if you pleaſe) after this Method, *viz.* Let the Quote have as many Decimal Places as you annexed o's, wherein if it comes ſhort, let the Defect be ſupplied with as many o's as is the Difference prefix'd. Or thus, if one o being added, the Numerator is leſs than the Denominator, ſet a o after the Decimal Point in the Quote; and if, another o being added, it is ſtill leſs, ſet another o in the Quote, and thus proceed by placing o's in the Quote 'till you come to the firſt ſignificant Figure, and carry on the Diviſion to what Degree of Exactneſs you pleaſe. *Vide* Exa. 4 and 5 following.

Exa. 1. Reduce $\frac{1}{8}$ to a Decimal.

Say 8 : 1 : : 1000 : .125 *Anſwer.*

Or rather thus,

8) 1.000 (.125 *Anſwer.*
8
—
20
16
—
40
40
—

Exa. 2. Reduce $\frac{3}{4}$ to a Decimal.

4) 3.00 (.75 *Anſwer.*
28
—
20

Exa. 3. Reduce $\frac{6}{7}$ to a Decimal.

7) 6.0000000

.8571428, *&c.* the Remainder being 4.

Exa. 4. Reduce $\frac{2}{25}$ to a Decimal.

25) 2.00 (.08 *Anſr.*

200

Exa. 5. Reduce $\frac{1}{476}$ to a Decimal.

476) 1.00000 (.0021008

952

480

476

4000

3808

192

In Exa. 3. there remains 4, ſo the Quote wants $\frac{4}{7000000}$ of its complete Value, that is $\frac{6}{7} = .8571428 + \frac{4}{7000000}$. In Exa. 5. there remains 192, which, according to the Rule for valuing the Remainder, is $\frac{192}{1000000}$, ſo that the Quote wants $\frac{192}{476000000}$ of its complete Value. And ſo of others.

Becauſe all Common Fractions cannot be reduced to their equivalent Decimals without a Remainder (for every Number is not an aliquot Part of another, elſe there could be no Remainder) it is neceſſary to ſhew, how far the Redn&tion is to be carried, ſo that its correſponding Decimal ſhall not want an Unit of the loweſt Species, nor any aſſignable Fraction of its true Value.

RULE. Conſider to what Integer or Unit refers; and if it has as many Places (after the Point) as that Integer has Figures when reduced to its loweſt Species, then the Decimal found does not want an Unit of the loweſt Species. For Exa. If the Decimal of 1 *L.* has three Places after the Point, it does not want $\frac{1}{1000}$, and conſequently does not want 1 *far.* which is greater, *viz.* $\frac{1}{960}$ *L.* of 1 *L.* In like manner, if the Decimal of

of a C. is carried to six Places after the Point, it cannot want $\frac{1}{1000000}$ of 1 C. consequently; it cannot want 1 qr. of a Dram, which is greater, *viz.* $\frac{1}{114688}$ of 1 C.

Now if a Decimal is to be multiplied by any Number (suppose of 6 Places) that the Product may not want an Unit of the lowest Species, let the Decimal be carried to as many Places after the Point, as is the Sum of the Figures of the Multiplier + the Number of Figures of the lowest Denomination, that make an Unit of the Denomination, of the Decimal. So if the Decimal of 1 *L.* was to be multiplied by a Number of 6 Places, let the proposed Decimal be carried to 9 Places (before you begin to multiply) and the Product shall not want 1 *far.* of the Truth. And so of others. Now that the Quote may not want any assignable Fraction of its true Value, carry the same to as many Places as is the Number of 0's in the Decimal Denominator, of the assigned Fraction. Thus, let $\frac{3}{7}$ be reduced to a Decimal, which shall not want $\frac{1}{10000000}$ of the Truth.

```
7) 3.00000 ( 42857
   28
   --
    20
    14
    --
     60
     56
     --
      40
      35
      --
       50
       49
       --
```

Here the Remainder is 1, which according to the Rule is $\frac{1}{1000000}$, consequently the Quote wants $\frac{1}{7000000}$, which is less than $\frac{1}{1000000}$.

In like manner, if the Decimal of 1 *L.* must not want $\frac{1}{1000000}$ of a *far.* let the Quote be carried to 8 Places, and it shall not want the proposed Fraction.

The Reaſon is; if it be carried to 3 Places it cannot want a *far*. therefore if it is carried 5 Places further, then it is carried to as many Places as the Denominator of the propoſed Fraction has o's, and conſequently cannot want that Fraction.

In dividing a whole Number by another, if there happens to be a Remainder, it is oftentimes convenient (eſpecially if the Quote is to be multiplied) to annex o's, ſetting a Point immediately after the Integral Part of the Quote, and to continue the Diviſion as far as is neceſſary, the Figures found in the Quote, by this Addition of o's to the Remainders, being all Decimals, and the more there are of them, the Quote is the nearer to be complete.

Exauples.

58) 3682 (63.4827	24) 7827 (326.125
348	72
202	62
174	48
280	147
232	144
480	30
464	24
160	60
116	48
440	120
406	120
Rem. 34	

In Exa. 1. there remains 34, which is .0034, ſo that the Quote wants $\frac{34}{10000}$ of its complete Value.

In Exa. 2. after the Integral Diviſion is over, there remains 3, to which annexing a o, and dividing, I place 1 (after a Point) in the Quote, and there remains 6; to

to which I annex another 0, and ſet 2 in the Quote; laſtly there remains 12, wherefore I annex another 0, and ſetting 5 in the Quote, the ſame is compleat without any Remainder.

In like manner, if the Dividend is a ſimple applicate Number, and the Diviſor abſtract, inſtead of reducing the Remainder to the next known Denomination, and dividing, you may carry on the Diviſion decimally, and the Decimal Part of the Quote is equivalent (or nearly ſo) to theſe inferiour Species, which would have been found by reducing the Remainders, and carrying on the Diviſion vulgarly. Take an Exa. done both Ways. What is the 7th Part of *L.* 248 ?

Vulgarly.	Decimally.
7)248 (35 *L.*	7)248 (35.42857
21	21
38	38
35	35
3	30
20	28
7)60 (8 *ſh.*	20
56	14
4	60
12	56
7)48 (6 *d.*	40
42	35
6	50
4	49
7)24 (3 *far.*	1
21	
3	

The

The Value of the Decimal .42857 (as ſhall be taught preſently) is 8 *ſh.* 6 *d.* 3 *far.* and ſomewhat more, as by the other Method.

Note, This Queſtion may be ſhorter wrought, thus,

Vulgarly.	Decimally.
7)248	7)248
35: 8: 6: 3 3/7	35.42857, *&c.*

2. In moſt Caſes, tho' you ſhould continue the Diviſion to ever ſo many Places in the Quote, there will ſtill be a Remainder, and the Quote will never be complete, becauſe there will happen a Remainder the ſame with the former; ſo that all the intermediate Figures in the Quote betwixt theſe two Remainders, will conſtantly be repeated, the Diviſion going on for ever. Such Decimals we call circulating or repeating Decimals, becauſe of the continual Return or Repetition of the ſame Figure or Figures; as alſo infinite, or indeterminate Decimals, becauſe the Diviſion can never come to an end, and conſequently the Decimal wants ſtill ſomething of the true Value of the Common Fraction. For Exa. 1/3 reduced to a Decimal is .333, *&c.* infinitely; for

3) 1.000
.333, *&c.*

Where the Dividend being always 10, the Quote muſt ſtill be 3. In like manner, 2/3 in a Decimal Expreſſion is .666 infinitely; becauſe the Dividend is always the ſame, for

3) 2.000
.666, *&c.*

Likewiſe 3/7 in a Decimal is .428571428571, *&c.* the Figures 428571 being continually repeated.

7) 3.000 (.4285714, here the Circulation begins,
28

20
14

60
56

40
35

50
49

10
7
---
30 the ſame with the firſt Dividual.
28

2

So that whenever the Quote begins to circulate, you need proceed no further in the Diviſion; but obſerving what Figure or Figures would ſtill be repeated, ſet down as many in the Quote as you pleaſe.

3. The Diviſion needs only to be continued ſo far, that the Defect may be inconſiderable, and the Remainder neglected as of no Value, the Method of which has been already taught, *p.* 124.

CASE II.

To reduce a Decimal of an higher Species to an equivalent Decimal of a lower.

RULE. Multiply the given Decimal by ſuch a Number of Units as makes 1 of your Decimal Denomination, and that Product by ſuch a Number of Units as makes 1 of the laſt Denomination, and ſo on till you come to the loweſt required, minding to point off

off as many for Decimals in the laſt Product of all, as there were Places in the given Decimal.

Exa. 1. Reduce .005 *L.* to the Decimal of 1 *d.*

.005
20

Anſr. 1 : 2 *d.*

0100
12

Here the Anſwer is a mixt Number, or an Improper Decimal.

1.200

Exa. 2. Reduce .05 *L.* to the Decimal of 1 *d.*

.05
20

Anſr. 12 *d.* Here the Anſwer is Integral.

100
12

12.00

Exa. 3. Reduce .153 C. to the Fraction of 1 *L.*

.153
4

612
28

Anſr. 17.136 C.

4896
1224

17.136

CASE III.

To reduce a Decimal of an inferiour Species to an equivalent of an higher.

RULE. This Caſe being the Converſe of the laſt, muſt require a contrary Operation, and therefore divide here as you multiplied there.

Exa. 1. Reduce .125 *far.* to the Decimal of 1 *L.*

4)

4) .125 *f.*
12) .03125 *d.*
20) .002604, *&c. ſh.*
.0001302, *&c. Anſr.*

Exa. 2. Reduce .847 *d.* to the Decimal of 1 *L.*

12	.847 *d.*
20	.07058333, *&c. ſh.*
	.00352916, *&c. Anſr.*

Exa. 3. Reduce .16 oz. to the Decimal of 1 qr. of 1 C.

28	16	.16 oz.
	4	.01 lib.
	7	.0025
		.00035714, *&c. Anſr.*

CASE IV.

To reduce an Integer of a lower Species or Denomination, to an equivalent Decimal of an higher Species.

RULE 1. If it is a ſimple Number, turn it firſt into a Common Fraction, and then reduce that Common Fraction to a Decimal by this Chap.

Exa. 1. Reduce 7 *ſh.* to the Decimal of 1 *L.*
7 *ſh.* is $\frac{7}{20}$ *L.* therefore, 20) 7.0
.35 *L. Anſr.*

Exa. 2. Reduce 3 *far.* to the Decimal of 1 *L.*
3 *far.* is $\frac{3}{960}$, therefore,

960) 3.0000 (.003125 *L. Anſr.*
2880
1200
960
2400
1920
4800
4800

Exa.

Exa. 3. Reduce 5 oz. to the Decimal of 1 *L.*
5 oz. is $\frac{5}{16}$ lib. therefore,

16) 5.00 (.3125 lib. *Anſr.*
48

20
16

40
32

80

Exa. 4. Reduce 1 oz. to the Decimal of 1 C.
1 oz. is $\frac{1}{1792}$ of 1 C. therefore,

1792) 1.00000 (.000558, *&c.* C. *Anſr.*
8960

10400
8960

14400
14336

64

Exa. Reduce $\frac{9}{16}$ *d.* to the Decimal of 1 *L.*
$\frac{9}{16}$ *d.* is $\frac{9}{16}$ of $\frac{1}{12}$ of $\frac{1}{20} = \frac{9}{3840}$ *L.* therefore,

3840) 9.0000 (.00234375 *L. Anſr.*
7680

13200
11520

16800
15360

14400
11520

28800
26880

1920
1920

But

But ſuch Reduction is ſooner made by dividing gradually from one Species to another, as ſhall be illuſtrated by repeating the preceding Examples.

Exa. 1.

20) 7.00
.35 *L.*

Exa. 2.

4 | 3.000
12 | .750
20 | .0625
.003125 *L.*

Exa. 3.

16 } 4 | 5.000
4 | 1.250
.3125 *L.*

Exa. 4.

16 } 4 | 1.000000
4 | .250000
28 } 4 | .062500
7 | .015625
4 | .0022321
.0005580 *&c.* C.

Exa. 5.

16 } 4 | 9.00000
4 | 2.25000
12 | .56250
.20) .0468875
.002344375

2. But if the Number to be reduced is mixt, that is, if it conſiſts of ſeveral Denominations, then firſt reduce each of them to the Decimal of the Denomination required, and add theſe together for the Anſwer. Or, ſecondly, reduce the mixt Number to a ſimple one of the loweſt Species mentioned, and thence into a Decimal.

Or, thirdly, reduce the Number of the loweſt Species to the Decimal of the next ſuperiour (whether there is any Number of that ſuperiour Species in the Queſtion or not) and to it add the Number of that Species in the Queſtion (if there is any) and reduce the Sum to the next higher Species, adding to the Decimal laſt found, the Number of that Species given in the Queſtion, and ſo go on till you come to the deſigned Integer.

Exa. 1. Reduce 7 *ſh.* 11 *d.* 1 *f.* to the Decimal of 1 *L.* By the firſt Method I work thus; 7 *s.* is $\frac{7}{20}$ *L.* $=.35$ *L.* next 11 *d.* is $\frac{11}{240}$ *L.* $=.04583333$ *&c.* *L.* and laſtly 1 *far.* is $\frac{1}{960}$ *L.* $=.001041666$ *&c.* Their Sum is .39687499 (or .396875, becauſe the two laſt Figures are 9's) for the Anſwer.

By the ſecond Method I reduce the 7 *ſh.* 11 *d.* 1 *f.* to *far.* and they make 381, which is $\frac{381}{960}$ *L.* = .396875 *L.* as before.

By the laſt Method I work thus; 1 *far.* is 25 *d.* to which I add 11 *d.* making the Sum 11. 25 *d.* This I divide by 12 and find .9375 *ſh.* to which adding the 7 *ſh.* I divide the Sum by 20, and the Quote is .396875 *L.* as before. And ſo of others.

4	1.00000	
12	*11.25000	
20	* 7.93750	
	.396875	

Note 1. The Numbers taken in, are marked with an Aſterisk. 2. The eaſieſt and exacteſt of theſe Methods I take to be the laſt, becauſe of the great Trouble that ariſes from the Reduction and Diviſion in the two firſt Methods, eſpecially the firſt, where there are ſo many Diviſions.

In like manner, 1 *qr.* 19 *lib.* 5 *oz.* 10 *dr.* reduced to the Decimal of 1 C. is .422781808, thus

16	4		10.000000 *dr.*
	4		2.500000
16	4	*	5.625000 *oz.*
	4		1.406250
28	4	*	19.35156250 *lib.*
	7		4.837890625
4		*	1.691127232 *qrs.*
			.422781808 C. *Anſr.*

But there are ſeveral other Methods for ſuch Reductions, as 1ſt in general, by Decimal Tables of Money, Weight and Meaſure, *&c.* framed and conſtrued for the purpoſe, ſhewing the Decimal of any inferiour Species in the Denomination of any ſuperiour (of the ſame kind) which Tables, tho' commonly met with in Books on this Subject, I ſhall not inſert; ſave that of Money, becauſe they are ſeldom uſed.

Decimal

Decimal Table of Money.

	Integer 1 *L.*			Integer 1 *L.*
	1	.0010416—	*ſh.* 1	.05
	2	.0020833—	2	.1
	3	.003125	3	.15
Penny.	1	.0041666—	4	.2
	2	.0083—	5	.25
	3	.0125	6	.3
	4	.016—	7	.35
	5	.02083—	8	.4
	6	.025	9	.45
	7	.02916—	10	.5
	8	.03333—	11	.55
	9	.0375	12	.6
	10	.04166—	13	.65
	11	.04583—	14	.7
			15	.75
			16	.8
			17	.85
			18	.9
			19	.95

Note. The Sign — annexed to ſome of the Decimals, ſignifies that they are too little, and not exactly equivalent to their correſponding Integers.

2. More particularly, by finding the Decimal of an Unit of any inferiour Species in the Denomination of any ſuperiour (when it can be found without a Remainder, which more ſeldom happens) and uſing the ſame as a Standard for finding the Decimals of the reſt of that inferiour Species. Thus the Decimal of 1 *ſh.* in the Denomination of 1 *L.* being found to be .05; any Number of Shillings multiplied by it gives their Decimal Value; for Example, 9 *ſh.* reduced to the Decimal of 1 *L.* this way is 45 *L.* for $9 \times .05 = .45$. And ſo of others.

But there is another eaſier Method of reducing Shillings to Decimals of 1 *L.* thus: If the Number of Shillings is even, take its Half, and ſet a Point before it; if odd, ſuppoſe a 0 annexed, and then take its Half. Thus 6 *ſh.* is .3 *L.* and 13 *ſh.* is .65 *L.*

In like manner, the Decimal of 1 Penny in the Denomination of 1 *L.* being .004166, *&c.* any Number of Pence multiplied hereby gives their Decimal Value in the Denomination of 1 *L.* but becauſe this Standard is incomplete, we muſt make ſome Allowance when multiplying, according to the Figures multiplied.

Alſo the Decimal of 1 *far.* in the Denomination of 1 *L.* being .00104166, *&c.* this multiplied by 2 gives .00208333, *&c.* for the Decimal of 2 *far.* And ſo on.

After the ſame manner you may make up Tables for all our common Weights and Meaſures, having ſtill a due Regard to the Circulation of the Root (if the ſame happens to be indeterminate) yet a ſeparate Reduction, tho' more tedious, is the ſafeſt way.

CASE V.

To find the Value of a Decimal Fraction in Integers of known Species.

RULE. Multiply the given Decimal by the Number of Units of the next inferiour Denomination, as make an Unit of your Decimal Denomination; and then cut off ſo many Figures towards the Right of the Product, as there are Places in the Decimal; the Figures ſtanding to the Left of thoſe which are cut off, being ſo many Integers of that inferiour Denomination; and thoſe on the Right are the Remainder, which you muſt multiply by ſuch a Number of Units of the next inferiour Denomination, as make an Unit of the laſt Denomination, and point off as before, and thus go thro' all the Denominations, till you come to the loweſt.

Exa.

Exa. 1. What is the Value of

.546 *L.*
20

ſh. 10.920
12

d. 11.040

Anſw. 10 *ſh.* 11 *d.* $\frac{4}{100}$.

Exa. 2. What is the Value of

.65 *ſh.*
12

d. 7.80
4

f. 3.20

Anſr. 7 *d.* 3 $\frac{2}{10}$ *far.*

Exa. 3. What is the Value of

.48 *d.*
4

f. 1.92

Anſw. 2 *f.* ferè.

Exa. 4. What is the Value of

.384 C.
4

qr. 1.536
28

4288
1072

lib. 15.008
16

oz. 0.128
16

768
128

dr. 2.048

qr. lib. oz. dr.

Anſr. 1—15—0—2 $\frac{48}{1000}$.

Exa. 5. What is the Value of .4106 of a *Scotch* Gal.

.4106
8

Pints. 3.2848
4

Mut. 1.1392
4

Gills. 0.5568
2

Halfgills. 1.1136

pts. mut. gills. halfgills.

Anſr. 3—1—0—1$\frac{1136}{10000}$.

Exa. 6. What is the Value of .736 of a Year.

13

9.568
28

4544
1136

15.904
24

3616
1808

21.696
60

41.760

Mon. D. ho. min.

Anſr. 9—15—21—41$\frac{76}{100}$.

The Value of any Decimal of a Pound may be found, either by the Table or Inſpection; but on this I ſhall not inſiſt. See my Treatiſe of Fractions, Page 119, 120, 121.

CASE VI.

To reduce a Decimal Fraction to a common one.

RULE. Set the Decimal for a Numerator of a common Fraction; whoſe Denominator will be 1 with as many 0's as there are Figures in the Decimal: which common Fraction thus found, you may reduce to its loweſt Terms.

Exa. 1. .56 is $\frac{56}{100}$ or $\frac{14}{25}$ loweſt. *Exa.* 2. .038 is $\frac{38}{1000}$ or $\frac{19}{500}$. *Exa.* 3. .0648 is $\frac{648}{10000} = \frac{162}{2500}$ or $\frac{81}{1250}$ loweſt: And ſo of others.

CHAP.

CHAP. XIX. *The Rule of Three*

IS ſo called, becauſe therein are 3 Numbers given to find a fourth Proportional: tho' ſometimes there are more Numbers in the Queſtion, which are all ſuperfluous, as not entering into the Stating, but ſerving only to complete the Senſe. Theſe are always known by being repeated or mentioned oftener than once, either in the very ſame Words, or in others of the ſame Import and Signification.

This Rule being of moſt general Uſe in Arithmetical Calculations, I ſhall endeavour, by a Variety of choice Examples, to make it very plain and intelligible to the meaneſt Capacity; and ſhall be the more prolix and particular in it, as intending to caſt together here ſuch Queſtions as all other Authors on this Subject treat of, as belonging to particular Rules: Whereas it is plain they are nothing but a further Application of the Rule of Three. By theſe I underſtand what are commonly called the Rules of Intereſt, Diſcount, Barter, Exchange, Fellowſhip, Gain and Loſs, Tare and Trett, Alligation, *&c.* As for the Rules of Practice, and the Rule of Five Numbers, I ſhall treat of them by themſelves.

Now, as a great part of the Difficulty of this Rule lies in the right ſtating of the Terms, you are carefully to obſerve the following

DIRECTIONS.

Conſider what Name the thing ſought is of, *viz.* if it is required to find a Sum of Money, a Quantity of Meaſure, Weight or Time, *&c.* and ſet that Number or Term which is like it in the Queſtion middlemoſt: then take that Number on which the Demand lies, and

and place it on the Right, and the other (which belongs to the Suppoſition, and is always of the ſame Name, either general or particular, with the third or Right-hand Number) ſet on the Left. Thus the ſuperfluous Terms, if there are any, will preſently appear: The Terms being thus ſtated, you muſt reduce them all to ſimple Numbers, if they are mixt, and the two Extremes; that is, the firſt and third Numbers, to the ſame particular Denomination, if they are not ſo already. Then you are to conſider, whether the fourth Number ſought (or Anſwer to the Queſtion) ought to be greater or leſs than the middle Number. If it ought to be greater, you muſt multiply the middle Number by the greater Extreme; and divide the Product by the leſſer; but if the fourth Number, or thing ſought, muſt (according to the Senſe of the Queſtion) be leſs than the middle Number, this laſt is to be multiplied by the leſſer Extreme; and the Product divided by the greater. The Quote reſulting from this Diviſion is the fourth Term or Anſwer to the Queſtion in the ſame Name or Denomination, with the middle Number juſt when you multiplied, which if in a low one muſt be brought higher, to the more ready Comprehenſion of the Mind.

Note 1. When the firſt Term divides, the Queſtion is ſaid to be in direct Proportion; and when the third Term is Diviſor, it is called Indirect or Inverſe.

2. The middle Number is never uſed for Diviſor, but is always either multiplied by, or multiplies one of the Extremes, according to the Nature of the Queſtion.

3. When the firſt and ſecond Numbers are of, or can be reduced to the ſame Denomination, it is not neceſſary to reduce the third to the Denomination of the firſt, but the Anſwer will come out in the ſame Name with the third. *Vid. Qu.* 35.

4 After you have diſcovered which Term is Multiplier to the middle one, it is eaſieſt and ſhorteſt to multiply by the leſſer of the two.

5. When

5. When after Reduction the third Term is 1, the Queſtion is ſolved by Diviſion alone, without any Multiplication; and when the firſt Term is 1, it is ſolved by ſimple Multiplication without Diviſion. *Vide Queſt.* 2, 6, 9.

6. Many Queſtions in this Rule are complex, and require ſeveral previous Operations independent of one another, before you can come to a Proportion in order to a Solution. *Vide Queſt.* 20, 27, *&c.*

I have here purpoſely avoided that common Diſtinction of the Rule of Three into Direct and Inverſe, ſo much uſed by the Generality of the Authors in this Science, there being not the leaſt Neceſſity for ſuch Diſtinction to any Perſon who can take up the Meaning and Demand of a Queſtion propoſed. For is it not eaſy for one of common Senſe to know whether the Anſwer to a Queſtion ought to be greater or leſs than the middle Term? If it ought to be greater, it is plain the leſſer Extreme muſt be Diviſor to the Product of the other two; and if the fourth Term (according to the Senſe of the Queſtion) ought to be leſs, then it is as clear, that the Product of the other two muſt be divided by the greater Extreme. The Reaſon of both which is, that to find a greater Quote, or to make the Anſwer greater than the middle Term, we muſt take a leſs Diviſor than the Number we multiplied by; and to find a leſs Quotient, or to make the Anſwer leſs than the middle Number, we muſt take a greater Number for the Diviſor than is the multiplying Number; for the greater the Diviſor, the leſs is the Quote, and contrariwiſe.

Qu. 1. If 4 Yards of Cloth coſt 23 *ſh.* what is the Value of 15 Yards at the ſame rate?

State

	yds.	*ſh.*	*yds.*
State	4	23 ::	15
	15		
	115		
	23		
	4|345		
	20)86|5 : 3		
	L. 4 : 6 : 3		

ILLUSTRATION.

Here I multiply the middle Number by the greater Extreme 15, becauſe the Anſwer to the Queſtion muſt be greater, and dividing the Product by the firſt, there reſulteth 86 *ſh.* (the middle Number being *ſh.*) and 1 over which I reduce (mentally) to *d.* and it makes 12, which divided by 4 quotes 3 *d.* So the Anſwer is 86 *ſh.* 3 *d.* or *L.* 4 : 6 : 3

Qu. 2. What is the Value of an Ounce of any thing, when 8 *lib.* of the ſame coſt 19 *ſh.* 7 *d.* Sterling?

	lib.	*ſh.*	*d.*	*oz.*
State	8 :	19—	7 : :	1
	16	12		
	128	235 (1 *d.*		
		128		
		107		
		4		
	128)	428 (3 *f.*		
		384		
		44		

Here I reduce the 8 *lib.* to *oz.* to correſpond with the third Number, alſo I reduce the middle Number to *d.* and after dividing by the firſt Term, (for it would have been unneceſſary to have multiplied by the third Term) the Quote is 1 *d.* and 107 *d.* remaining, which I reduce to *f.* and dividing as before, the Quote is 3 *f.* and 44 remaining; ſo the Anſwer is 1 *d.* $3\frac{44}{128}$ *f.* or $\frac{11}{32}$ loweſt.

Q. 3. If I buy 5 Yards of Cloth for *L.* 1—4, how many Yards at the ſame Rate may I have for *L.* 29—10—3?

State

L.	*s.*	*yds.*	*L.*	*s.*	*d.*
State 1	—4	: 5 : :	29	—10	—3
20			20		
—			—		
24			590		
12			12		
—			—		
288			7083		
			5		

288) 35415 (122 *yds.*
288
661
576
855
576
279
4
) 1116 (3 *qrs.*
864
252
4
) 1008 (3 *nails.*
864
144

Here I reduce the Remainder 279 to *qrs.* and divide again by 288, and there remains 252, which I reduce to *Nails*, and dividing again, the total Anſwer is 122 *yds.* 3 *qrs.* 3 *nails*, and $\frac{1}{2}$ *nail* for $\frac{144}{288} = \frac{1}{2}$.

Qu. 4. If 7 Men ſpend *L.* 37—15—6 in 12 Days, how long will *L.* 119—5 ſerve them at the ſame rate of ſpending?

State

days.

State *L.* 37—15—6 : 12 : : *L.* 119—5

20 | 20

755 | 2385

12 | 12

9066 | 28620

12 *mid. numb.*

9066) 343440 (37 *Days.*

27198

71460

63462

7998

24

9066) 191952 (21 *ho.*

18132

10632

9066

Here the 7 Men is the ſuperfluous Term, for the Anſwer would have been the ſame, whatever Number of Men had been mentioned, ſince the Suppoſition and Demand are equally affected by it.

Qu. 5. Of what Weight ought the Penny Loaf of Bread to be, when the Peck of Wheat is ſold for 1 *ſh.* 6 *d.* if I get 8 *oz.* for a Penny, the Wheat being at 2 *ſh.*

ſh. *oz.* *ſh.* *d.*

State 2 : 8 : : 1—6

12 | 12

24 *d.* | 18 *d.*

8

18 } 3 | 192

6 | 64

10 „ 13 „ 8

Here the ſuperfluous terms are 1 Loaf and 1 Peck: alſo I conſider that I'll have more Weight for my Money when the Wheat is cheaper, therefore I multiply the middle Number by the greater Extreme, or

or which is the ſame, this by that; ſo the Anſwer is, 10 *oz.* 13 *d.* wt. 8 *qr.* (Troy Weight.)

Qu. 6. If a Nail of Velvet coſt 16 *d. Sterling*, what is the Value of 75 Yards 3 qrs. at the ſame rate?

n. d. yds. qrs.
State 1 : 16 : : 75—3
4
303
4
1212
16
12) 19392
20)1616
L. 80—16 *Anſr.*

Here the firſt Term being 1, dividing by it is unneceſſary, and therefore the Anſwer is found in Pence by Multiplication only; which Pence I reduce to *L.* by dividing by 12 and 20.

Qu. 7. If I travel 600 Miles in 24 Days, when the Day is 12 Hours long, in how many Days, at the ſame rate of Travelling, ought I to finiſh the ſame Journey, when the Day is 16 Hours long (ſuppoſing in each that I travel from Sun-riſing to Sun-ſetting?)

ho. long. Days. ho. long.
State 12 : 24 : : 16
12
16 } 4 | 288
4 | 72
18

Here the ſuperfluous Term is 600 Miles, and I multiply the middle Number by the leſſer Extreme; becauſe the Anſwer muſt be leſs than the 2d Term, ſo the Quote is 18 Days.

Qu. 8. What time will 10 Men take to do a Piece of Work, when 4 other ſuch could do the ſame in 15 Hours, 12 Minutes?

Men. ho. min. Men.
State 4 : 15—12 : : 10
4

10)60—48
6— $4\frac{4}{5}$

Here I multiply the middle Number by the leſſer Extreme, becauſe 10 Men will take leſs time. The ſuperfluous Term is 1 Piece of Work. Obſerve alſo, that I don't reduce the middle Number, becauſe the Multiplier is ſmall.

Qu. 9. How much will *L.* 1—6—3 *per* Day amount to in a Year?

D. L. s. d. *D.*
State 1 : 1—6—3 : : 365
20 315

26 1825
12 365
1095
315

12)114975
20)9581—3
479—1—3

In a Year are 365 Days. The 1ſt Term being 1, the Queſtion is performed by pure Multiplication, and the Anſr. comes out in *d.* which reduced makes *L.* 479—1—3.

Qu. 10. How many C. of Sugar, at $6\frac{1}{2}$ *d. per* Pound, may I purchaſe for *L.* 43—18—4?

d. lib. *L. s. d.*
State $6\frac{1}{2}$: 1 : : 43—18—4
2 20

13 878
12
10540
2

13) 21080
28 } 4 | 1621 $\frac{7}{13}$
7 | 4054
4157—25 $\frac{7}{13}$
Anſr. 14—1—25 $\frac{7}{13}$

Here I reduce the Extremes to Halfpence, and the Anſr. is 1621 $\frac{7}{13}$ lib. which reduced makes 14 C. 1 qr. 25 $\frac{7}{13}$ lib.

Qu.

Qu. 11. What principal Sum will gain *L.* 5 in 7 Months, when *L.* 100 gains the ſame in a Year, or 12 Months?

	Mon.	*L.*	*Mon.*
State	12	: 100	: 7
	12		
	7)1200		
	L. 171—8—$6\frac{6}{7}$		

Here 7 Months will require a greater Principal, becauſe the time is ſhorter. The ſuperflu. Term is *L.* 5.

Qu. 12. If $47\frac{3}{4}$ Yds. of 6 qr. broad Cloth hang a Room, *&c.* how many Yds. of Yard broad Cloth will do it? Or rather thus, How many Yds. of Yard-broad Cloth are equal preciſely in Content to $47\frac{3}{4}$ Yds. of 6 qrs. broad?

	qrs.	*yds.*	*qrs.*
State	6	: $47\frac{3}{4}$	: : 4
	6		
	4)286—2		
	71—2—2		

Here I conſider that there muſt be taken more in length of the narrow Cloth, to equalize that of the broader; therefore I multiply by the greater Extreme; and the Anſr. is 71 Yds. 2 qrs. 2 nails. This Queſtion may be wrought ſhorter, by adding $\frac{1}{2}$ of 47—3 to itſelf; thus

2)47—3
23—3—2

71—2—2 *Anſr.* as before.

Qu. 13. If 560 Men, being in Garriſon, have Proviſion only for 3 Months, how many of the ſaid Number muſt be turned out, that the Proviſion may ſerve the remaining Men other 5 Months?

	Mon.	*Men.*	*Mon.*
State	3	: 560	: : 8
	3		
	8)1680		
	210		

Here 3 Mon. and other 5 make 8 Mon. and conſidering that fewer can be maintained 8 Mon. on the ſame Quantity, than

than can be done for 3 Mon. I multiply by the leſſer Extreme.

560
210 to be kept in.

Anſr. 350 to be turned out.

Qu. 14. If when Wheat is 6 *ſh.* 5 *d.* the Buſhel, the Penny-Loaf weighs 12 *oz.* what ought the Wheat to be ſold at, when the Loaf weighs 17 *oz.* 10 *d.* weight?

	oz.	*ſh.*	*d.*		*oz.*	*d. wt.*
State	12	: 6—	5	: :	17—	10
	20	12			20	
	240	77			350	
		240				
		308				
		154				

Here I conſider that the Wheat muſt be cheaper when the Loaf weighs more, therefore the Proportion is indirect.

35 | 0 } 5 | 184 | 10
7 | 369—2 ½
12 | 52—3 7/35 or 1/5
ſh. 4—4—3 1/5 *Anſr.*

Qu. If 8 Men do a piece of Work in 6 Days, in what time will they do 16½ times as much?

	W.	*D.*		*W.*
State	1	: 6	: :	16½
				6

99 Days *Anſr.*

Q. 16. What is the Intereſt of *L.* 37—13—4 for a Year, when *L.* 7 is the Intereſt of *L.* 100 for the ſame time?

State

State *L.* 1|00 : 7 : : *L. ſh. d.* 37—13—4

7

2|63—13—4

20

12|73

12

8|80

4

3|20

Here inſtead of dividing by 100 the long way, I cut off the two o's, and two Figures for them, reducing the Remainder, and the Anſr. is *L.* 2—12—8—$3\frac{1}{5}$

The ſame may be wrought thus

100 : 7 : : 37—13—4

7

100 } 10 | 266—13—4
10 | 26— 7—4

2—12—8—$3\frac{1}{5}$ as before.

Qu. 17. If a Servant's daily Wages be 6*d.* 1 *far.* how much will it amount to, being forborn 3 Years, 8 Months, and 10 Days?

State *D. d. y. m. D.*
1 : $6\frac{1}{4}$: : 3—8—10

13

47
28

376
95

1326 Days.
3 Days add.

1329 Days in all.
$6\frac{1}{4}$

In a Year are 13 Months and 1 Day, therefore I add 3 Days for the 3 Years.

7974
332

12|8306¼
2|0)69|2—2¼
L. 34—12—2¼ *Anſr.*

Qu. 18. If I receive 30 Stone 12 oz. of Soap for a Debt of *L.* 8—7—4, how much doth it ſtand me *per lib?*

	St.	*lib.*	*oz.*	*L.*	*s.*	*d.*	*lib.*
State	30	0	12 :	8	7	4 : :	1
	16			20			16
	480			167			16 *oz.*
	16			12			
	7692 *oz.*			2008			
				16			

7692)32128(4 *d.*
30768
1360
4
5400(0 *far.*

d. f.
Anſr. 4—0 $\frac{13}{19}\frac{60}{23}$ *per lib.*

Qu. 19. A owes B *L.* 117—10—2 and compounds the Debt by paying 17 *ſh.* 4 *d. per L.* how much ought B to receive according to this Diſcount, and how much doth he loſe?

	L.	*s.*	*d.*	*L.*	*s.*	*d.*
State	1 :	17	4 : :	117	10	2
	20	12		20		
	20	208		2350		
	12			12		
	240			28202		

Here I ſay, if 1 *L.* fall to 17 *ſh.* 4 *d.* how much will the Sum due fall to?

240 28202 208

225616
56404

24 | 0 } 4 | 586601 6
6 | 146650 ½
12 | 24441 $\frac{176}{240}$
2|0)20316—9
L. 101—16—9 $\frac{176}{240}$ = $\frac{11}{15}$

From 117—10—2
Take 101—16—9 $\frac{11}{15}$ receives.
15—13—4 $\frac{4}{15}$ loſes.

Qu. 20. What will 3 Pieces of Holland Cloth come to, the firſt Piece containing 15 $\frac{1}{4}$ Yds. the ſecond 19 $\frac{1}{2}$, and the third 10 $\frac{3}{4}$ at 17 *ſh.* 4 *d.* for 3 Yards?

15 $\frac{1}{4}$
19 $\frac{1}{2}$
10 $\frac{3}{4}$
45 $\frac{1}{2}$ yds. in all.

State half yds.

yds.	*ſh.*	*d.*	*yds.*
3 :	17—	4 : :	45 $\frac{1}{2}$
2	12		2
6	208		91 half yds.
	91		
	208		
	1872		
6)	18928		
12)	3154—2 $\frac{2}{3}$		
2|0)	26|2—10—2 $\frac{2}{3}$		

L. 13—2—10—2 $\frac{2}{3}$ *Anſr.*

Or without reducing the Extremes to half yards, thus

yds.	*d.*	*yds.*
3 :	208 : :	45 $\frac{1}{2}$
	45 $\frac{1}{2}$	
	1040	
	832	
	104 = $\frac{1}{2}$ of 208	
3 |	9464	

3154—2 $\frac{2}{3}$ = *L.* 13 : 2 : 10 : 2 $\frac{2}{3}$ as before

Qu.

Qu. 21. A Gentleman has an Estate of *L.* 1460—10—4 *per Annum*, how much does he lay by yearly, spending daily at the rate of *L.* 1—18—6?

	D.	*L.*	*sh.*		*D.*
State	1	: 1—	18½	: :	365
		20			38½
		38½			2920
					1095
					182½
					2l0) 1405l2½
					L. 702--12--6

Here I first find how much he spends yearly, which subtracted from his yearly Income gives the Answer.

From	1460—10—4	yearly Income.
Take	702—12—6	yearly Expences.
	757—17—10	lays by. *Answ.*

Qu. 22. What is the Interest of *L.* 313—11—6 for a Year and 74 Days at *L.* 5—10 *per Cent. per annum?*

	L.	*L.*	*L.*	*sh.*	*d.*
State	1l00	: 5½	: : 313—	11—	6
					5½
			1567—	17—	6
			156—	15—	9
			17l24—	13—	3
			20		
			4l93		
			12		
			11l19		

Here I first find its Int for a Year, next for 74 Days, and the Sum of both is the Answ.

Again,

D. L. ſh. d. D.
Again, 365 : 17—4—11 :: 74
20

344
12

4139
74

16556
28973 12|
365) 306286 (839
2920 2|0) 619—11
1428 3—9—11
1095

3336
3285

51

	L. ſh. d.
Int. for a Year	17—4—11
Int. for 74 Days	3—9—11
Anſr.	20—14—10

But ſuch Queſtions are more exactly wrought by one Operation, as you'll ſee further on.

Qu. 23. How many Ducats at 7 *ſh.* 4 *d. per* Piece, are equal in Value to 300 Dollars, at 4 *ſh.* 8 *d. per* Dollar?

Dol. ſh. d. Dol.
1 : 4—8 :: 300
12

56
300

16800 Value of the Doll.

Here I firſt find the Value of the Dollars, and from, that I have the Value of the Ducats, by

ſaying

ſh. 4 *Duc.* *d.*
ſaying 7—4 : 1 : : 16800
12

88) 16800 (190
88

800
792

Rem. 80 *d.*

Anſr. 190 *Duc.* and 80 *d.* over and above.

But all Queſtions of this nature are better ſolved by one Operation, thus

ſh. d. *ſh. d.*
4—8 : : 300 : 7—4
12 12

56 88
300

88) 16800 (190
88

800
792

Rem. 80

Here I conſider, that there muſt be fewer Pieces at 8 *ſh.* 4 *d.* taken to equalize a Number of other Coin at 4 *ſh.* 8 *d.* therefore multiply by the leſſer Extreme. Take another Exa. wrought after this method.

Qu. 24. How many lib. of Tea, at 9 *ſh.* 6 *d. per* lib. muſt B give A for 22 Reams of Paper, at 5 *ſh.* 10 *d. per* Ream?

ſh. *d.* *ſh.* *d.*
State 5—10 : 22 : : 9—6
12 12

70 114
22

114) 1540 (13 *lib*
114

400

So that B muſt give A 13 lib. of Tea, and 58 *d.* more for his 22 Ream, at 5 *ſh.* 10 *d. per* Ream.

400

400
342

Rem. 58 *d.*

Qu. 25. Lent upon Bond *L.* 546 on the 18th *Jan.* 1737, what will be due at *Whitsunday* 1739, the Rate of Intereſt being at 6 *per Cent. per annum*?

Betwixt 18th *Jan.* 1737 and 18th *Jan.* 1739 are 2 Years = 730 Days; and betwixt 18th *Jan.* and 15th *May* are 117 Days, thus found,

		L.	*L.*	*L.*
State		1|00 :	6 : :	546
				6
				L. 32|76
				20
				ſh. 15|20
				12
				d. 2|40
				4
				f. 1|60

	Jan.	13	*Days.*	*L.*	*ſh.*	*d.*	*f.*
	Feb.	28					
	March	31					
	April	30					
	May	15					
		117					
		730					
		847					

Again 365 : 32—15—2—1 : : 117

20

655
12

7862
4

31448
117

220136 220136

220136
31448
31448

365) 3679416 ($\frac{4}{1}$0080
365 12)2520
2|0)21|0
2941 10—10
2920

216

Anſr. L. 622 : 0—4½

L. 32—15—2—1 Intereſt for 1 Year.
2

65—10—4—2 Intereſt for 2 Years.
10—10—0—0 Intereſt for 117 Days.

76—0—4—2 Total Intereſt
546—0—0—0 Principal lent.

But this, like Qu. 22. can be wrought by one Operation.

Qu. 26. In *L.* 4176—11—8 *Scotch*, how much *Sterling*?

L. Sc. *L. St.*
State 12 : 1 : : 4178—11—6
12 4178—11—6
Anſr. 348— 4—3½

Here I have no more to do but to divide by 12, becauſe 12 *L. Scots* are equal to 1 *L. Sterling.*

Qu. 27. In 42 Bags of a certain Commodity, weighing Groſs 146 C. 2 qrs. 24 lib. Tare 7 lib. *per* 112 lib. and Trett 4 lib. *per* 104, how many lib. neat?

112
7 ſubtract

112 : 105 :: 146—2—24

4

586
28

4692
1174

16432
105

82160
16432

112 { 4 | 1725360
4 | 431340
7 | 107835

15407 *lb. Suttle*

Anſr. 14812½ *lib.* Nett.

Again

104
4 ſub.

104 : 100 :: 15405

15405

104 | 1540500 (14812
104

500
416

845
832

130
104

260
208

52

This Method is ſomewhat tedious. *Vide* Page 169, where you'll find a ſhorter.

Qu. 28. A and B make a joint Stock. A puts in *L.* 127, and B *L.* 31: In Trading they gain or loſe *L.* 16: What is each proportional Share of the ſame?

A 127
B 31

158 Sum of the Stocks

State 158 *l.* : 16 *l.* :: 127 *l.*

16

762
127

158) 2032 (12 *l.*
158

452
316

136
20

) 2720 (17 *s.*
158

1140
1106

34
12

) 408 (2 *d.*
316

92
4

368 (2 *f.*
316

52

Again 158 *l.* : 16 *l.* : : 31 *l.*

31

16
48

) 496 (3 *l.*
474

22
20

) 440 (2 *s.*
316

124
12

) 1488 (9 *d.*
1422

66
4

) 264 (1 *f.*
158

106

A 12—17—2—2 $\frac{82}{158}$
B 3— 2—9—1 $\frac{100}{158}$

Proof 16— 0— 0—0

Here I add the Stocks together for the first Term, then I say, If the whole Stock gain or lose so much, what will each particular Stock gain or lose? But Questions of this nature are capable of a more compendious Solution. *Vide* Page 187.

Qu.

Qu. 29. A, B, and C bought a Parcel of Cloth containing in all 748 Yards, for *l.* 212, whereof A paid *l.* 187, B 110, and C the reſt. How much of the Cloth belongs to each?

	L.	*Yds.*	*L.*		*Yds. Qrs. Nails.*
State	412 :	748 ::	187		339—2—0 $\frac{32}{412}$
	412 :	748 ::	110	to the *Anſrs.*	199—2—3 $\frac{140}{412}$
	412 :	748 ::	115		208—3—0 $\frac{240}{412}$
					Proof 748—0—0

Vide Page 187.

Qu. 30. If a Pound of any thing is bought for 16 *ſh.* 4 *d.* and ſold for 17 *ſh.* 6 *d.* how much is gained on *l.* 187?

17—6
16—4

16—4 : 1—2 :: 100
12 12
196 14
100

196) 1400 (7 *l.*
1372
28
20

196) 560 (2 *ſh.*
392
168
12

196) 2016 (10 *d.*
196
56

By Subtraction I find 16 *ſh.* 4 *d.* gains 1 *ſh.* 2 *d.* therefore I ſay how much will *l.* 100 gain? where I don't reduce the *l.* 100. *Vid.* Note 3. Page 149.

Qu. 31. What will *l.* 54 amount to, being forborn 15 Years, at 6 *per Cent. per annum*, ſimple Intereſt?

State 100 : 6 : : 54

6

3|24

20

4|80

12

5|60

4

2|40

l. *s.* *d.* *f.*

3—4—9—2$\frac{3}{5}$

15

48—12—0—0 Int.

54—0—0—0 Principal.

102—12—0—0 Amount.

Here I find the Intereſt of of *l.* 54 for a Year, which multiplied by 15 gives the Intereſt for 15 Years, which added to the Principal makes *l.* 102—12 for the Amount.

Qu. 32. A, B, and C purchaſe a Ship for *l.* 3000, whereof A paid *l.* 1400, B 1000, and C the reſt. Her Freight for the firſt Voyage was *l.* 374, what muſt each Partner receive of the ſame proportionally to his Share in the Purchaſe?

A 1400
B 1000
C 600
3000

30|00 : 374 : : 14|00

14

1496

374

30 } 5|5236
   } 6|1047—4

174—10—8

3|000 : 374 : : 1|000

3|374

124—13—4

30|00 : 374 : : 6|00

6

30 } 5|2244
   } 6| 448—16

74—16

A

A	174	10	8
B	124	13	4
C	74	16	0
Proof	374	00	0

Qu. 33. A Man is indebted to A, B, and C the following Sums, *viz.* to A *L.* 400, to B *L.* 500, and to C *L.* 700, but is only worth *L.* 979. How much of the same must each receive, and what Loss do they sustain severally?

A 400
B 500
C 700

16|00 :: 976 :: 4|00
4

16 { 4|3904
4| 976
244

16|00 : 976 :: 5|00
5

16 { 4|4880
4|1220
305

{ .61×400=244
{ .61×500=305
{ .61×700=427

Or shorter thus, by finding the Proportional Share of *L.* 1.

1600)976(.61 Then

16|00 : 776 :: 7|00
7

16 { 4|6832
4|1708
427

Then by subtracting what each receives, from what he ought to have received, you'll find A loses *L.* 156, B *L.* 195, and C *L.* 273.

Qu. 34. A has 40 Yards of Cloth at 8 *sh.* 4 *d. per* Yard, ready Money, but in Barter he will have 10 *sh.* 2 *d.* B hath Wool at 20 *d. per lib.* ready Money; How much Wool must be delivered to A for his Cloth, and how much is the Price of the Wool to be raised in Barter?

Yd. ſh. d. *Yds.* *d. lib. d.*

1 : 8—4 : : 40 Then 20 : 1 : : 4000

12 1

100 210)4000

40 200 lib. of Wool.

4000 *d.* Value of the Cloth in ready Money.

ſh. d. *ſh. d.* *d.*

Laſtly, 8—4 : 10—2 : : 20

12 12

100 122 *d. f.*

20 24—1 $\frac{6}{10}$ Value of the Wool *per lib.* in Barter.

2440

4

1160

After finding the Value of the Cloth at the ready Money Price, I ſay, If 20 *d.* buy 1 *lib.* of Wool, how much will the Value of the Cloth buy? *Anſ.* 200 *lib.* Then to find how much the Price of the Wool is to be raiſed in Barter, I ſay, If 8—4 ready Money riſe to 10 *ſh.* 2 *d.* in Barter, what will 20 *d.* ready Money riſe to? And the Anſwer is, 24 *d.* 1 $\frac{6}{10}$ or $\frac{3}{5}$ *far.*

Qu. 35. What is the Diſcount of *L.* 542 : 14 : 8 for 3 Months at 5 *per cent. per annum?*

M. L. M. *L. ſh.* *L. ſh.* *L. ſh. d.*

State 12 : 5 : : 3 101—5 : 1—5 : : 542—14—8

3 20 20 20

12)15 2025 25 10854

1—5 Int. of 12

L. 100 for 3 Months. 130256

130256

Brought over ——— 130256

25

651280

260512

3256400

2025 { 5 | 651280
5 | 130256
9 | 14472
9 | 1608 200/2025

12)1608

20)1311—0

6—14—0 200/2025 or 8/81

Here, and in all Queſtions of this Nature, I muſt firſt find the Intereſt of *L.* 100 for the Time, and at the Rate propoſed, which adding to *L.* 100, I ſay, if that Aggregate loſe the Intereſt of *L.* 100 already found, what will the Sum propoſed loſe? and the Quote is the Anſwer required; which taken from the Sum propoſed, leaves the Sum to be paid preſently; and to find this at once (without ſubtracting) let *L.* 100 be the middle Term in the former Stating.

	L.	*ſb.*	*L.*	*L.*	*ſb.*	*d.*
Thus	101	—5 :	100	:: 542	—14	—8
	20		20	20		
	2025		2000	10854		
				12		
				130256		
				2000		

5)2025

5)405

9)81

9)9

1

So that 2025 is = 5×5×9×9.

2025 { 5 | 260512000
5 | 52102400
9 | 10420480
9 | 1157831

12)128647 1825/2025 or 73/81

Brought

Brought over 12|128647 $\frac{182}{20}\frac{4}{25}$ or $\frac{73}{81}$

2|0)1072|0—7$\frac{73}{81}$

L. 536—0—7$\frac{73}{81}$ *Anſ.* as by the firſt Method: For,

	L.	*ſh.*	*d.*
From	542	—14	—8
Take	6	—14	—0$\frac{8}{81}$
Rem.	536	—00	—7$\frac{73}{81}$

Vide a ſhorter Method, *Appendix, of ſimple Intereſt*, Tab. I.

Qu. 36. If I buy a Yard of Cloth at 3 *ſh.* ready Money, and ſell it immediately at 3 *ſh.* 4 *d.* with eight Months Credit, what ſhould I gain if I ſhould ſell *L.* 100 worth at the ſame Rate, with twelve Months Credit?

ſh.	*d.*	*L.*		*M.*	*L.*	*ſh.*	*d.*	*M.*
State 3	: 4	:: 100	Then	8	: 11—02—		2$\frac{2}{3}$	:: 12
12		4					8	
36	36 { 6 6	400		12|38—	17—	9$\frac{8}{3}$		
		66—13—4		*L.* 7—	08—	1$\frac{7}{9}$ or 1$\frac{1}{9}$ *ſ.*		
	L.	11—02—2$\frac{2}{3}$						

By the firſt Operation I find, that if 3 *ſh.* gain 4 *d.* in any Time, *L.* 100 in the ſame Time will gain *L.* 11—2—2$\frac{2}{3}$; then I ſay, If 8 Months gain ſo much, 12 Months will gain leſs, (in this Caſe) becauſe the longer Goods are credited, the Seller has the leſs Profit. But this may be wrought by one Operation.

Qu. 37. Bought 40 *doz.* Pair of Stockings for *L.* 42, how may I ſell them *per doz.* to gain *L.* 15 *per cent.*

L. *L.* *L.* *Doz.* *L.* *ſh.* *Doz.*

State 1|00 : 115 :: 42 Then 40 : 48—06 :: 1

42 1

230

460

L. 48|30

20

ſh. 6|00

40 { 5|48—06
8| 9—13—22

L. 1—04—1$\frac{32}{40}$ or 3$\frac{1}{5}$ *f.*

Qu. 38. A Farmer mixes 8 Pecks of Wheat at 16 *d.* *per* Peck, with 9 Pecks at 18 *d.* with 12 Pecks at 17 *d.* what is a Peck of the Mixture worth?

P. *d.* *d.*

8 at 16 = 128

9 at 18 = 162

12 at 17 = 204

29 494

P. *d.* *P.*

Then 29 : 494 :: 01

1

29)494(17$\frac{1}{29}$ *d.* *Anſ.*

29

204

203

1

If it was required what 2, 3 or 4, *&c.* Pecks would coſt, make, 2, 3, or 4, *&c.* the laſt Term (inſtead of 1 here, and you'll have the Anſwer.

Qu. 39. What is the Compound Intereſt of *L.* 387—10—0 forborn 3 Years at 6 *per Cent. per annum*, and what is the Amount?

State

State *l.* 100 : 6 :: 387—10
6
23|25—00
20
5|00
Again 100 : 6 :: 410—15
6
24|64—10
20
12|90
12
10|80

387—10—00 1st Principal.
23—05—00 1st Year's Int.
410—15—00 2d Principal.
24—12—10 2d Year's Int.
435—07—10 3d Principal.
26—02—05 3d Year's Int.
461—10—03 Amount *Anſ.*

L. L. L. ſh. d.
Laſtly, 100 : 6 :: 435—7—10
6
26|12—7—0
20
2|47
12
5|64

L. 23-05-00
24-12-10
26-02-05
Sum of Int. 74-00-03
Prin. lent 387-10 00
Amount 461-10-03 as before.

So you ſee I always add the Intereſt to the Principal, which Sum becomes a new Principal, and have as many Operations, as is the Number of Years the Money lies out. See *Chap.* 2. *Tab.* IV. *of compound Intereſt*, where you'll find a ſhorter Method.

Qu. 40. A and B entered into Company, A put in *L.* 55, which continued in Stock 4 Months, and B put in *L.* 131 for 7 Months; they gained or loſt *L.* 48, what is each Man's Share in the ſame?

55×4=220
131×7=917

1137

L. L. L.
1137 : 48 :: 220
48

1760
880

1137)10560(9 *l.*
10233

327
20

1137)6540(5 *ſh.*
5685

855
12

1137)10260(9 *d.*
10233

27

L. L. L.
Again 1137 : 48 :: 917
917

7336
3668

1137)44016(38 *l.*
3411

9906
9096

810
20

)16200(14 *ſh.*
1137

4830
4548

282
12

1137)3384(2 *d.*
2274

1110

A L. 9—05—9 $\frac{27}{1137}$
B 38—14—2 $\frac{1110}{1137}$

Proof 48—00—0

Here

Here I multiply each's Money by the Time it continues in Stock, the Sum of both which is the first Term, the Gain or Loss the second Term, and the Product of each's Stock by his Time the third Term, in two distinct Operations.

Vide Pag. 187, where you'll find a more compendious Method.

Qu. 41. A hath 36 *lib.* of Galls at 11 *d. per lib.* ready Money, in Barter 12 *d.* 81 Yards of Cloth at 16 *d. per* Yard ready Money, in Barter 17 *d.* 54 Reams of Paper at 5 *sh.* 10 *d. per* Ream ready Money, and in Barter 6 *sh.*

B has Wine at 13 *sh. per* Gallon, Silk at 15 *sh. per* Yard, and Brandy at 29 *sh. per* Cask, } ready Money;

How many Gallons of Wine, Yards of Silk and Casks of Brandy, of each a like Number, must B give A for his Goods, advancing his own proportionally?

36 *lib.* at 11 *d.* = 396
81 Yards at 16 *d.* = 1296
54 Reams at 70 *d.* = 3780

5472 *d.* Total Value of A's Goods at the ready Money Price.

13 *sh.*
15 *sh.*
29 *sh.*

57 *sh.* or 684 *d.* Value of 1 of each of B's Things at the ready Money Price. Then I say, If 684 *d.* buy 1 of each of B's, how many of B's Things will the total Value of A's Things buy?

d. *of each.* *d.*
684 : 1 :: 5472
1

684)5472 (8 of each, *Answ.*
5472

Qu.

Qu. 42. In 36 C. 1 qr. 12 *lib.* groſs, how much nett Weight, Tare at 8 *lib. per* 112 *lib.* and Trett 4 *lib. per* 104? *C. qr. lib.*

$14\left\{\begin{matrix}2\\7\end{matrix}\right.$ | 36 : 1 : 12
18 : 0 : 20
2 : 2 : 10 : 13 $\frac{5}{7}$

26 | 33 : 3 : 01 : 02 $\frac{2}{7}$ Suttle
1 : 1 : 05 : 06 $\frac{7}{91}$

Anſ. 32 : 1 : 11 : 11 $\frac{46}{91}$ Nett.

Here I uſe a ſhorter Method than that of *Qu.* 27. for I conſider that 8 *lib.* is $\frac{1}{14}$ of 112 and 4 *lib.* is $\frac{1}{26}$ of 104.

Therefore, if the Tare is at 2 *lib. per* 112 *lib.* ſubtract $\frac{1}{56}$, if at 4 *lib.* ſubtract $\frac{1}{28}$, if at 6 *lib.* ſubtract $\frac{1}{28} + \frac{1}{2}$ of that $\frac{1}{28}$; if at 14 *lib.* ſubtract $\frac{1}{8}$, *&c.*

Qu. 43. My Friend lent me *L.* 46 : 15, for 5 Months, and he having occaſion afterwards to borrow from me *L.* 100, I demand how long he ought to uſe it in order to a full Requital of his former Kindneſs?

L.	*ſh.*	*M.*	*L.*
State 46—	15 :	5 : :	100
20			20
935			2000
5			

2|000)41575(2 *M.*
41

675
30

)20250(10 *Days.*

M. Days.

Anſ. 2 : 10

Here I conſider that *L.* 100 being a greater Sum, muſt not lie out ſo long as *L.* 46—15, therefore I multiply the middle Number by the leſſer Extreme, and divide by the greater.

Qu. 44. A Merchant is to receive *L.* 500 *Sterling*, either in Dollars at 4 *ſh.* 4 *d. per* Piece, which are worth but 4 *ſh.* 3 *d.* or in *French* Crowns at 6 *ſh.* 1 $\frac{1}{2}$ *d.*

per Piece, which are worth but 6 *ſh.* which of theſe ought he to take, in order to ſuſtain the leaſt Loſs?

	ſh.	*d.*	*d.*	*ſh.*	*d.*
4 : 4	4	— 4 :	1 : :	6	— $1\frac{1}{2}$
4 : 3	12			12	
1	52			73	
	2			2	
	104			)147(	$1\frac{43}{104}$
				104	
				43	

So that in receiving the Dollars, each 6-$1\frac{1}{2}$ loſeth only $1\frac{43}{104}$; whereas, if he ſhould take the Crowns, every 6—$1\frac{1}{2}$ loſeth $1\frac{1}{2}$ *d.* which is greater than $1\frac{43}{104}$, and therefore he ought to take the Dollars.

Qu. 45. If 35 Ells at *Vienna* make 24 at *Lyons*, and 3 Ells at *Lyons* make 5 at *Antwerp*, and 100 at *Antwerp* make 125 at *Frankfort*, how many Ells at *Frankfort* make 42 at *Vienna*?

	Ant.	*Frank.*	*Ant.*	*Frank.*
1. If 100	:	125	: : 5	: $6\frac{1}{4}$
	Ly.		*Ly.*	
2. If 3	:	$6\frac{1}{4}$	: : 24	: 50
	Vien.		*Vien.*	
3. If 35	:	50	: : 42	: 60 *Anſr.*

Or thus:

35 *V.* = 24 *L.*
3 *L.* = 5 *A.*
100 *A.* = 125 *F.*
42 *V.* = ?

Then 24×5×125×42=630000 for a Dividend; and 35×3×100=10500 for a Diviſor.

Wherefore 10500)630000(60 Anſwer, as before.
630
0

Qu. 46. If 35 Ells at *Vienna* make 24 at *Lyons*, and 3 Ells at *Lyons* make 5 at *Antwerp*, and 100 Ells at *Antwerp* make 125 at *Frankfort*, how many at *Vienna* make 60 at *Frankfort*?

L.

	L.	V.	L.	V.
1. If	24 :	35 ::	3 :	$4\frac{3}{8}$
	A.		A.	
2. If	5 :	$4\frac{3}{8}$ ::	100 :	$87\frac{1}{2}$
	F.		F.	V.
3. If	125 :	$87\frac{1}{2}$ ::	60 :	42 *Anſr.*

Or thus:

35 *V.* = 24 *L.* Then 35×3×100×60=630000 for
3 *L.* = 5 *A.* a Dividend; and
100 *A.* = 125 *F.* 24×5×125=15000 for a Diviſor.
60 *F.* = ?

Wherefore 15|000)630|000(42 Anſwer as before.
60 And ſo of others, whether they conſiſt of more or fewer Terms.
30
30

Qu. 47. A and B made a joint Stock, A put in *L.* 300, and at the End of 3 Months took out *L.* 75, and at the End of 7 Months after put in *L.* 50.

B put in at firſt *L.* 180 for 5 Months, and then put in *L.* 48 more, and two Months after that took out *L.* 40. At the End of 12 Months they find *L.* 100 of Gain. How much of the ſame is due to each?

L. 300 for 3 Months is 900
75 Subtr.

225 for 7 Months is 1575
50

275 for 2 Months is 550

L. 3025 Sum of A's Stocks into his Times.

L. 180 for 5 Months is 900
48 Add

228 for 2 Months is 456
40 Subtr.

188 for 5 Months is 940

2296 Sum of B's Stocks into his Times.

A 3025
B 2296

5321 Sum of both.

Then 5321)100.00(.0187934, *&c.*

Wherefore 3025x.0187934=56.850035=A's Share.
And 2296x.0187934=43.149646=B's Share.

Proof 99.999681=100 *L. fere.*

The Reaſon why it makes not *L.* 100, is becauſe the Decimal .0187934 is incomplete.

Qu. 48. If I pay for 45 Pints of Brandy *L.* 4: 10, how muſt one Pint be ſold to gain *L.* 10 *per Cent?*

100 : 108 :: 4.5 : 4.86 Price of the whole 45 Pints at the propoſed Gain. Then to find the Price of 1 Pint,

45 { 5 | 4 86
9 | .972

.108 *L.* 2 *ſh.* 1 *d.* $3\frac{17}{25}$ *far. Anſr.*

Qu. 49. How ought *L.* 350 to be divided among 3 Men, A, B, and C, ſo as A may have $\frac{1}{2}$, B $\frac{2}{3}$, and C $\frac{1}{4}$?

$\frac{1}{2}+\frac{2}{3}+\frac{1}{4}=\frac{12+16+6}{24}=\frac{34}{24}$ Here the Sum of the Parts is greater than the whole: And therefore 'tis impoſſible to give them the Parts propoſed; but the Meaning of the Queſtion muſt be to divide *L.* 350 in ſuch Proportion as $\frac{1}{2}$, $\frac{2}{3}$ and $\frac{1}{4}$ bear to one another. Wherefore, ac-

neglecting the Denominator 4, I find a common Multiplier thus: 34)350(10.29412+

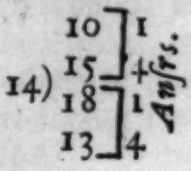
Then 10.29412×12=123.52944=123 : 10 : 7

And 10.29412×16=164.70592=164 : 14 : 1½ *ferè.*

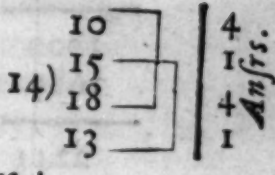
Alſo 10.29412× 6= 61.76472= 61 : 15 : 3½

L. 350 : 0 : 0 Proof.

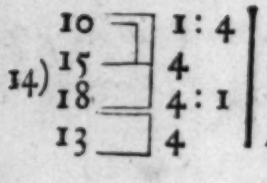
Qu. 50. A Merchant, mingles Tobacco thus; He takes ſome at 10 *per lib.* ſome at 15 *d. per lib.* ſome at 18 *d. per lib.* and ſome at 13 *d. per lib.* how many *lib.* muſt he take of each Sort to ſell a *lib.* of the Mixture at 14, ſo as neither to gain nor loſe?



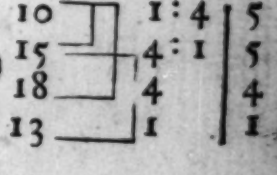
or thus:



Or thus:



or thus:



Or thus:

So that there are 5 different Anſwers, and all true.

Here, and in all Queſtions of this Nature, you muſt ſet the ſimple Prices under one another, and connect them ſo together, as that one leſſer and one greater than the mean Price may be always joined together, and then take the Difference betwixt the mean Price and that of each ſimple, ſetting the ſame over againſt the ſimple wherewith it ſtands connected, and the ſeveral Differences are the Anſwers ſought.

Qu. 51. How many Crowns at 56 5/6 *d.* are equal to *L.* 600 : 12 : 8 *Sterling?*

d. *Cr.* *L.* *ſh.* *d.*

State $56\frac{1}{6}$: 1 : : 600 — 12 — 8

6 20

337 12012

12

144152

6

337)864912(2566 *Cr.*

674

1909

1685

2241

2022

2192

2022

170

3

)510(1 *Livre.*

337

173

20.

)3460(10 *ſols.*

337

90

Cr. *Liv.* *Sols.*

Anſr. 2566 : 1 : 10

CHAP.

CHAP. XX. *Contractions in the Rule of Three Numbers, otherwiſe called Rules of Practice.*

CASE I.

WHEN the Value of an Unit is an Aliquot Part of any ſuperior Species, or equal to the Sum of two or more Aliquot Parts; alſo if it is a Part or Parts of ſuch Aliquot Parts, take that Part, or Parts of the Number given, whoſe Value is ſought, and you have the Anſwer in the ſame Name with that ſuperior Species, reducing the Remainder, if any be, and finding its Value.

TABLE I. *Of the Aliquot Parts of* 1 L. Sterl.

ſ.	d.			
1	0	is a	20th	of 1 L. Sterling.
1	8		12th	
2	0		10th	
2	6		8th	
3	4		6th	
4	0		5th	
5	0		4th	
6	8		3d	
10	0		2d	

Of the Aliquant or uneven Parts of 1 L.

3—0	is	$\frac{1}{10}+\frac{1}{20}$
6—0		$\frac{1}{5}+\frac{1}{10}$ or $\frac{1}{4}+\frac{1}{20}$
7—0		$\frac{1}{4}+\frac{1}{10}$
8—0		$\frac{2}{5}$
9—0		$\frac{1}{5}+\frac{1}{4}$

11—0

11—0
12—0
13—0 } is { 1/2 + 1/20 ; 1/5 ; 1/5 + 1/20 ; 1/5 + 1/10
14—0
15—0
16—0
17—0
18—0
19—0 } is { 1/4 ; 1/4 + 1/20 ; 1/4 + 1/10 ; 1/4 + 3/20 ; 1/4 + 1/5

TABLE II. *Of the Aliquot or even Parts of* 1 ſh.

d. f.		
1—0	is a	12th
1—2		8th
2—0		6th
3—0		4th
4—0		3d
6—0		2d

of 1 Shill.

Of the Aliquant or uneven Parts of 1 ſh.

d. f.		
5—0	is a	1/3 + 1/12 ; 1/4 + 1/3 ; 2/3 ; 3/4 ; 3/4 + 1/12 ; 3/4 + 1/6
7—0		
8—0		
9—0		
10—0		
11—0		

of 1 Shill.

TABLE III. *Of the Aliquot Parts of* 1 d.

1, 2, 3 } is { 1/4, 1/2, 3/4 } of 1 Penny.

Examples of this firſt CASE.

Exa. 1. 480 Yards at 1 *ſh. per* Yard. 1 *ſh.* is 1/20 *l.*
Therefore 20)4810
24 *L. Anſr.*

Here

Here I conſider that 480 Yards at *L.* 1 *per* Yard would make *L.* 480, therefore at 1 *ſh. per* Yard, it muſt be only the 20th Part of what it was formerly, becauſe 1 *ſh.* is he 20th Part of 1 *L.* And this accounts for the Method of operating this and the ſubſequent Examples.

Exa. 2. 376 *lib.* at 4 *ſh. per lib.*

5)376

L. 75—4 *Anſr.*

4 *ſh.* is $\frac{1}{5}$ *l.* and the 1 over, after Diviſion is $\frac{1}{5}$ *l.* or 4 *ſh.*

Exa. 3. 538 *oz.* at 2 *d. per oz.*

6)538

20)89—8

L. 4—9—8 *Anſr.*

2 *d.* is $\frac{1}{6}$ *ſh.* therefore I divide by 6, and the Anſwer is 89 *ſh.* and 4 over, which is $\frac{4}{6}$ *ſh.* or 8 *d.* The *ſh.* I reduce to *l.* by dividing by 20.

Exa. 4. 95 Stone Weight at 6 *ſh.* 8 *d. per* Stone.

3)95

L. 31—13—4

6 *ſh.* 8 *d.* is $\frac{1}{3}$ *l.* After Diviſion there remains 2, which is $\frac{2}{3}$ *l.* or 13 *ſh.* 4 *d.*

Exa. 5. 120 *lib.* Weight at 1 $\frac{1}{2}$ *d. per lib.*

8)120

15 *ſh. Anſr.*

1 $\frac{1}{2}$ is $\frac{1}{8}$ *ſh.*

Exa. 6. 83 Yards at 3 *ſh.* 4 *d. per* Yard.

6)83

L. 13—16—8 *Anſr.*

3 *ſh.* 4 *d.* is $\frac{1}{6}$ *l.* after dividing there remains 5, which is $\frac{5}{6}$ *l.* or 5 times 3 *ſh.* 4 *d.* =16 *ſh.* 8 *d.*

Exa. 7. 315 Yards at 6 *ſh. per* Yard.

315

$\frac{1}{4}$ for 5 *ſh.* 78—15

$\frac{1}{5}$ of laſt for 1 *ſh.* 15—15 } Add

L. 94—10 *Anſr.*

6 *ſh.* is $\frac{1}{4}$ *l.* 4 $\frac{1}{20}$ *l.* or it is $\frac{1}{4}$ *l.* + $\frac{1}{5}$ of that $\frac{1}{4}$.

Exa.

Exa. 8. 413 *oz.* at 9 *d. per oz.*

413
3

4)1239
210)3019—9
L. 15—9—9 *Anſr.*

9 *d.* is $\frac{3}{4}$ *ſh.* wherefore I multiply by 3, and divide the Quote by 4.

Or I might have divided by 4 and then multiplied the Quote by 3, thus:

4)413
103—3
3

210)3019—9
L. 15—9—9

Exa. 9. 210 Ells at 15 *ſh.* 15 *ſh.* is 10 *ſh.* + 5 *ſh.*

210

$\frac{1}{2}$ for 10 *ſh.* - - 105
$\frac{1}{2}$ of laſt for 5 *ſh.* 52—10

L. 157—10

Exa. 10. 470 *lib.* at 11 *d.* 11 is $\frac{3}{4} + \frac{1}{6}$ *ſh.*

3

4)1410

352—6 = $\frac{3}{4}$
78—4 = $\frac{1}{6}$

210)4310—10
L. 21—10—10 *Anſr.*

Or the Anſwer may be found in *ſh.* by ſubtracting $\frac{1}{12}$, thus:

470

$\frac{1}{12}$ = 39—2

ſh. 430—10 as before.

Exa.

Exa. 11. 111 Ells at 11 *ſh.* 11 *d.* 3 *far. per* Ell.

111

½ for 10 *ſh.* -	55—10
$\frac{1}{10}$ of laſt for 1 *ſh.*	5—11
$\frac{1}{12}$ of 10 *ſh.* for 10 *d.*	4—12—6
$\frac{1}{10}$ of laſt for 1 *d.*	0— 9—3
½ of laſt for 2 *far.*	0— 4—7—2
½ of laſt for 1 *far.*	0— 2—3—3
	L. 66— 9—8—1 *Anſr.*

The Work is plain.

Exa. 12. 60 Stone Weight at 11 *ſh. per* Stone.

60

11 *ſh.* is ½ + $\frac{1}{20}$ *l.* or it is ½ *l.* + $\frac{1}{10}$ of ½.

½ for 10 *ſh.*	30
$\frac{1}{10}$ of laſt for 1 *ſh.*	3
L.	33 *Anſr.*

Exa. 13. 419 Yards at 19 *ſh. per* Yard.

20)419

20—19

19 *ſh.* is ¾ *l.* + ⅕ *l.* Or rather ſubtract $\frac{1}{20}$.

L. 398— 1 *Anſr.*

When the Price of an Unit is an Aliquot Part, ſuch Diviſion is certainly the moſt expeditious Way of ſolving the Queſtion: But when they are Aliquant or Compound Parts, the common Method of Reduction is preferable, as being ſhorter and leſs perplexed.

CASE II.

When the Price of an Unit is an uneven Number of *ſh.* multiply the Number whoſe Value is ſought, by the Half of the Shillings, doubling the firſt Figure or Unit's Place of the Product for *ſh.* the reſt are *L.*

Exa.

Exa. 1. What is the Value of 347 Yards at 8 *ſh. per* Yard?

347
4

L. 138—16 *Anſr.*

Exa. 2. What is the Value of 316 *lib.* at *L.* 1—12 *per lib?*

316
16

189—12
316

L. 505—12 *Anſr.*

L. 1—12 is 32 *ſh.* ½ of which is 16.

The Reaſon of this Method may be thus accounted for: To multiply by an even Number of *ſh.* and divide the Product by 20, is the ſame as to multiply by one half of the Shillings and divide the Product by 10: But to divide by 10 is the ſame as cutting off the Unit's Figure; thoſe that remain to the left being entire *L.* and the Figure ſo cut off being ſo many 10th Parts of a *L.* each of which is equal to 2 *ſh.* wherefore the Figure ſo cut off, muſt be doubled or multiplied by 2.

CASE III.

When the Price of an Unit conſiſts of *L. ſh.* and *d.* or *L.* and *ſh.* firſt multiply the Number given by theſe *L.* and for the *ſh.* and *d.* take one of the preceding Caſes. The Sum is the Anſwer.

Exa. 1. What is the Value of 16 C. at *L* 4—6—8 *per* C?

16
4

64
⅓ of 16 5—6—8

L. 69—6—8 *Anſr.*

Here I firſt multiply by 4, and for 6 *ſh.* 8 *d.* I take ⅓ of the Number given, and their Sum is the Anſwer.

Exa.

Exa. 2. 413 *lib.* at *L.* 1—16—8 *per lib.*

at 1 *l.*	413
½ for 10 *ſh.*	206—10
½ of laſt for 5 *ſh.*	103— 5
⅓ of laſt for 1 *ſh.* 8 *d.*	34— 8—4
	757— 3—4 *Anſr.*

Exa. 3. 149 C. at *l.* 2—8—6 *per* C.

2

298

⅕ of laſt for 8	59—12
1/16 of laſt for 6 *d.*	3—14—6
L.	361— 6—6 *Anſr.*

CASE IV.

When that Extreme; which is the Multiplier, is an Aliquot Part or Aliquot Parts of the Unit, which is Diviſor, take ſuch Part or Parts of the middle Number, and their Sum is the Anſwer.

Exa. 1. What is the Value of 7 *lib.* when 1 Quarter of a C. coſts *l.* 43—10?

4)43—10

L. 10—17—6 *Anſr.*

7 *lib.* is ¼ Quarter.

Exa. 2. What is the Value of 1 Quarter 2 Nails at 18 *ſh. per* Yard.

18

¼ for 1 Quarter,	4—6
⅛ for 2 Nails	2—3
ſh.	6—9 *Anſr.*

Here I take ¼ for 1 Quarter, and ⅛ for 2 Nails, and the Sum 6 *ſh.* 9 *d.* is the Anſr.

Exa. 3. What is the Value of 4 *lib.* at *l.* 1—15—6 *per* Stone?

4 | 1—15— 6

0— 8—10—2 *Anſr.*

4 *lib.* is ¼ of 1 Stone.

CASE V.

When after a Queſtion in the Rule of Three is ſtated, and reduced according to the Directions already given, that Extreme, which is Diviſor, and any of the other two are diviſible by the ſame Number, divide both by that Number, and work with the Quotes in their ſtead. By this means the Diviſor and that other Term will be reduced lower, and conſequently the Work eaſier; and ſometimes one of them will be reduced to 1, ſo that the Operation will be performed by a ſimple Multiplication or Diviſion. This Rule holds good whether the Proportion be direct or inverſe.

Exa. 1. What is the Value of 20 Yards of Cloth, when 8 Yards coſt *L.* 6—12—8?

State 4)8 : 6—12—8 :: 4)20
Quote 2 : 5 5 Quote

2|13 3—03—4
16—11—8 *Anſr.*

Here I divide the Diviſor 8, and the other Extreme 20 by 4; ſo that the ſtating after this Reduction is the ſame as to ſay, If 2 Yards coſt *L.* 6—12—8, what will 5 Yards coſt?

Exa. 2. If 12 *lb.* buy 8 Yards of Ribbon, how many Yards may I have for *L.* 4—4 at the ſame Rate?

lb. *yds.* *l.* *ſh.*
State 4)12 : 4)8 :: 4—4
20

3 : 2 :: 84
2

3)168
56 *Anſr.*

Here, after reducing the third Term to *ſh.* I divide the Diviſor 12 and the middle Number 8, each by 4, and ſo the Stating is, If 3 *lb.* buy 2 Yards, how many Yards will 84 *lb.* buy?

Cr,

Or, being it is the ſame, which of the other 2 Terms I divide by that Number which meaſures the Diviſor, I divide the Diviſor 12 and the other Extreme, 84, each by 12, and the Operation is ſhorter yet.

Thus: 12)12 : 8 :: 12)84
1 : 8 :: 7
7

56 *Anſr.* as before.

Exa. 3. How long will 45 Men take to do a Piece of Work, when 9 other ſuch could do it in 18 Hours?

	M.	*Ho.*	*M.*
State	9) 9	: 18 ::	9) 45
	1	: 18 ::	5
	1		
	5) 18		
	Ho. 3—36 *Anſr.*		

Here I divide the Diviſor 45 and the other Extreme; each by 9, the Quotes are 5 and 1; ſo that the Stating is reduced to this Form, If 1 Man take 18 Days, how long will 5 Men take?

Or I might have divided 45 and 18 by 9, and the reduced Stating would have been

M.	*Ho.*	*M.*
9	2 ::	5
9		
5) 18		
3—36 *Anſr.* as before.		

Exa. 4. If I buy 400 Yards for 120 *L.* how much will 1200 Yards coſt at the ſame Rate?

Yds.	*L.*	*Yds.*
State 4|00	: 120 : :	12|00
4)4	: 120 : : 4)	12
1	: 120 : :	3
3		
360		

Here I divide the Diviſor and the other Extreme; 1200, each by 100, (or which is the ſame Thing) I cut off two o's from each, and they are reduced to 4 and 12, then I divide each of theſe by 4; ſo that the Queſtion becomes the ſame as if I ſhould ſay, If 1 Yard coſt 120 *L.* what will 3 Yards coſt? The ſame Reſult would have been, if the Diviſor 4 and the middle Term had been abbreviated, by dividing both by 40.

Thus: 40)400 : 40)120 : : 1200

Second State 10 : 3 : : 1200

Then by dividing the Diviſor 10 and the other Extreme; both by 10, the State would be 1 : 3 : : 120, the Reſult of which is a ſimple Multiplication, as before.

CASE VI.

When a Queſtion is to be ſolved by ſeveral Operations of the Rule of Three: Numbers, and the two firſt Terms in each Operation are the ſame, divide the ſecond Number by the firſt, and the Quote will be a common Multiplier, by which you muſt multiply all your third Numbers, and theſe Products are the Anſwer.

Exa. Let it be propoſed to divide 500 *L.* amongſt 4 Men, A, B, C and D, ſo that as oft as A has 1 *L.* B may have 3; and as oft as B has 3 *L.* C may have 5 *L.* and as oft as C has 5 *L.* D may have 6 *L.* how much muſt each receive?

By

By the common Method, the Proportions are,

1	16 : 500 :: 1 :	31— 5	}	= 500 Proof.
3	16 : 500 :: 3 :	93—15		
5	16 : 500 :: 5 :	156— 5		
7	16 : 500 :: 7 :	218—15		
16				

But by the contracted Method I work thus:

16)500(31.25

Then 31.25×1=31.25 Alſo 31.25×5=156.25
And 31.25×3=93.75 Laſtly 31.25×7=218.75

I ſhall not further inſiſt on theſe Contractions, leſt I ſhould rather perplex than inſtruct the Learner, and indeed I'm of Opinion that thoſe ſix Caſes already mentioned, comprehend the moſt, if not all that can be pertinently ſaid on the Subject. I own, that if I ſhould follow the Method of ſome Authors, by beginning with the Price of an Unit at 1 *far.* and thus aſcending till I came to *L.* 1, I might make their Number much greater. Beſides, many of the Examples I have adduced (and innumerable others of the ſame Nature) can be ſooner wrought by the common Method of Reduction; and as there may be ſeveral Ways of working one and the ſame Queſtion, the chuſing the ſhorteſt and eaſieſt depends entirely on Practice and Experience. 'Tis true, when the Price of an Unit is an Aliquot Part of the next ſuperior Species, (Caſe I.) there can be no ſhorter nor eaſier Method than what I have propoſed; but when it is an Aliquant, *viz.* a Compound Part, or Parts of Parts, the Trouble, by this Method, is conſiderably greater, and that by Reduction is preferable. So that when the Price is an Aliquant Part of *L.* 1, ſuch as 3, 6, 7, 8, 9, 11, *&c. ſh.* or of *ſh.* ſuch as 5, 7, 8, 9, 10, 11 *d.* I would in moſt Examples find the Anſwer by multiplying, and then reducing it to *L.* I now proceed to

CHAP. XXI. *Rule of Three in common Fractions.*

1. STATE the three Terms as in Integral Arithmetick; and perform the Work by multiplying and dividing, according to the Nature and Senſe of the Queſtion, obſerving in the Operation the Rules of Fractions already delivered.

2. After you have ſtated the three Terms as they are given in the Queſtion; if any of them is a whole or mix'd Number, or a compound Fraction, let it be reduced to a ſimple one.

3. If the Terms can be reduced lower, let that be done before you multiply, and ſo the Operation will be much facilitated, by having them in lower Expreſſions.

4. If the Extremes are not Fractions of the ſame Integer, reduce the one of them to the Denomination of the other, (and it ſeems moſt convenient that the Fraction of the lower Unit be reduced to an Equivalent Fraction of the higher.)

5. If you cannot readily know which of the Extremes is greateſt (and conſequently, at a Loſs how to multiply and divide) reduce them to one Denominator, and take the Numerators inſtead of the given Fractions, and work with them as with Integers.

6. If the Anſwer comes out in high Terms, or if it is an improper Fraction, let it be reduced; alſo, if you want the Value of it in known Species, find it by the laſt Caſe of Reduction of common Fractions.

Exa. 1. If $\frac{4}{5}$ of a Yard coſt $\frac{2}{7}$ *L.* what is the Price of $\frac{2}{3}$ of a Yard at the ſame Rate?

Yd. L. Yd.

State $\frac{4}{5} : \frac{2}{7} :: \frac{2}{3}$

$\frac{2}{7} \times \frac{2}{3} = \frac{4}{21}$, then $\frac{4}{5})\frac{4}{21}(\frac{5}{21}$ *L.* $= 4$ *ſh.* $9\frac{1}{7}$ *d. Anſr.*

Here $\frac{2}{3}$ Yard coſts leſs than $\frac{4}{5}$, therefore I multiply the middle Number by the leſſer Extreme; and divide the

the Product by $\frac{4}{1}$, and the Quote (in its loweſt Terms found by 5th Note of *Caſe* V. *Diviſ. Common Fractions*) is $\frac{5}{21}$ *L.* whoſe Value is found by *Caſe* XI. *of Reduction.*

Exa. 2. If $\frac{1}{3}$ *d.* buy $2\frac{3}{8}$ *oz.* How much will 4 *L.* buy?

d. oz. L.
State $\frac{1}{3} : 2\frac{3}{8} :: 4$
L. oz. L.
When reduced $\frac{1}{7\frac{1}{20}} : \frac{19}{8} :: \frac{4}{1}$
$\frac{19}{8} \times \frac{4}{1} = 7\frac{6}{8}$ then $\frac{1}{7\frac{1}{20}})7\frac{6}{8}$(6840 *oz.*
$= 427\frac{1}{2}$ *lib. Anſr.*

Here I reduce the $\frac{1}{3}$ *d.* to the Fraction of 1 *L.* the mixt Number $2\frac{3}{8}$ to a ſimple Fraction and the 4 *L.* to a Fraction; and after multiplying, I divide the ſhorteſt Way by 7th Note, *Caſe* V. *of Diviſion*, and the Anſwer is *oz.* which I reduce to *lib.* by 16.

Exa. 3. If of Cloth that is $1\frac{1}{4}$ Yard broad, $2\frac{3}{4}$ Yards will make a Coat, how much in length of another Cloth which is $\frac{3}{4}$ Yards in Breadth will make another Coat of the ſame Dimenſions? Or rather thus, How much of $\frac{3}{4}$ Yard broad Cloth is equal to $2\frac{3}{4}$ Yards of $\frac{5}{4}$ broad?

State $\frac{5}{4} : 1\frac{1}{4} :: \frac{3}{4}$
$5 \times 1\frac{1}{4} = 5\frac{5}{4}$ then $3)5\frac{5}{4}$($5\frac{5}{12}$
$= 4\frac{7}{12}$ *Yd.* or 4 *Yds.* 2 *Qrs.*
$1\frac{1}{3}$ *Nail.*

Here becauſe the Extremes have the ſame Denominator, I neglect it, and work with the Numerators.

Exa. 4. If $4\frac{1}{5}$ coſt $9\frac{1}{3}$ *d.* how much may be bought for $\frac{2}{9}$ *ſh.* at the ſame Rate?

d. lib. ſh.
State - - - $9\frac{1}{3} : 4\frac{1}{5} :: \frac{2}{9}$
By reducing the 2 firſt to Fractions, $\frac{28}{3} : \frac{21}{5} :: \frac{2}{9}$
ſh. *ſh.*
By reducing the firſt to the Fract. of 1 *ſh.* $\frac{28}{30} : 2\frac{1}{5} :: \frac{2}{9}$
Then by contracting the firſt Term, $\frac{7}{9} : 2\frac{1}{5} :: \frac{2}{9}$
And by neglecting the Denominators 9, $7 : 2\frac{1}{5} :: 2$
Therefore $2\frac{1}{5} \times 2 = 4\frac{2}{5}$ and $4\frac{2}{5} \div 7 = \frac{6}{5}$ *Anſr.* (by 4th Note *Caſe* V. *of Diviſion*) $= 1\frac{1}{5}$ *lib.*

Exa.

Exa. 5. What is the Value of $\frac{5}{8}$ Yard at 17 *ſh.* 9 *d.* *per* Yard?

Yd. *ſh.* *d.* *Yd.*

State 1 : 17—9 : : $\frac{5}{8}$
$\frac{8}{8}$: 17—9 : : $\frac{5}{8}$
8 : 17—9 : : 5
5

8)88—9
11—1$\frac{1}{8}$ *Anſr.*

Here I reduce the firſt Term to the Denominator of the third, and neglect the Denominators, becauſe they are the ſame in both Extremes. But in all Queſtions of this Nature, the Anſwer is ſooner found by multiplying the middle Number by the Number of the third Term, and dividing the Product by its Denominator, as you ſee it happens to be here, by means of the Method I have uſed in reducing.

Exa. 6. If $\frac{3}{4}$ *lib.* be worth 12 *ſh.* 7 *d.* what is the Price of 1 *lib.* at the ſame Rate?

State - - - $\frac{3}{4}$: 12—7 : : 1
By reducing the 3d Term to a Fract. having the ſame Denominator with the firſt Term, - } $\frac{3}{4}$: 12—7 : : $\frac{4}{4}$
By neglecting the Denominators, 3 : 12—7 : : 4
4

3)50—4
16—9 $\frac{1}{3}$ *Anſr.*

But in all Queſtions of this Nature, where the firſt Term is a Fraction and the laſt an Integer both of the ſame Name, the Anſwer is ſooner found by multiplying the middle Number by the Denominator of the Fraction, and dividing the Product by its Numerator.

Here follow ſome more Queſtions with their Anſwers, for the Learner's further Exerciſe in this Rule.

Qu. 1. In what Time will B perform a Piece of Work, when A alone can do it in 17 $\frac{1}{7}$ Days, and A and B together in 15 Days? *Anſr.* 120 Days.

Qu.

Qu. 2. Suppoſing 3 to be $\frac{1}{4}$ of 12, what would be $\frac{1}{4}$ of 20? *Anſr.* $3\frac{3}{4}$.

Qu. 3. What is the Value of $\frac{5}{16}$ *oz.* at $3\frac{3}{4}$ *d. per lib.*
Anſr. $\frac{75}{256}$ *far.*

Qu. 4. I bought at one time $19\frac{1}{7}$ *lib.* of a certain Commodity at $18\frac{2}{5}$ *d. per lib.* at another time I bought of the ſame Commodity $58\frac{4}{5}$ *lib.* at the Rate of $1\frac{1}{7}$ *d. per oz.* what came each to at their reſpective Prices; and which of them was the better Bargain, and by how much *per lib*?

Anſr. The Value of $19\frac{1}{7}$ *lib.* at $18\frac{2}{5}$ *d. per lib.* is *L.* 1—9—4—$0\frac{32}{35}$; the Value of $58\frac{4}{5}$ *lib.* at $1\frac{1}{7}$ *d. per oz.* is *L.* 4—9—0—$2\frac{50}{63}$, and the laſt is the cheapeſt Bargain by $\frac{4}{315}$ *d per lib.*

To extract the Roots of common Fractions.

RULE. Reduce them to their loweſt Expreſſions, and extract the Root of Numerator and Denominator, ſeverally. *Exa.* 1. What is the Square Root of $\frac{12}{27}$? This Fraction reduced to its loweſt Terms is $\frac{4}{9}$, the Root of which is $\frac{2}{3}$. *Exa.* 2. What is the Cube Root of $\frac{64}{343}$? The Cube Root of 64 is 4, and that of 343 is 7; ſo the Anſwer is $\frac{4}{7}$.

CHAP. XXII. *Rule of Three in Decimal Fractions.*

1. STATE the Queſtion in the ſame Terms wherein it is propoſed.

2. Reduce the common Fractions to Decimals; and if any of the Terms are mix'd, reduce the inferior Species to Decimals of the higheſt, annexing theſe Decimals, inſtead of the common Fractions or inferior Species

Species to their reſpective Integers (if there are any) with the Decimal Point betwixt them.

3. If the Extremes are not Fractions of the ſame Unit, let the one of them be reduced to the Denomination of the other.

4. The Preparatory Work being made, multiply and divide according to the Rules already delivered.

5. When the Quote is found, qualify it, that ſo you may know how much of it is Integral, and how much Fractional, and then you may reduce the Fractional Part to inferior Species of the Integer, and thus you will have the Anſwer complete, or nearly ſo, in a whole, mix'd, or fractional Number, according to the Nature of the Queſtion.

Exa. 1. If $\frac{3}{4}$ *d.* buy $\frac{1}{8}$ *oz.* what will $\frac{4}{5}$ buy at the ſame Rate?

State $\frac{3}{4} : \frac{1}{8} :: \frac{4}{5}$

Reduced to Dec. .75 : .125 :: .8

8

.75) 1.000 (.1333, *&c. oz. Anſr.*
75
250
225
250
225
250
225
25

By Com. Fract.
$\frac{1}{8} \times \frac{4}{5} = \frac{4}{40} = \frac{1}{10}$, then
$\frac{3}{4}) \frac{1}{10} (\frac{4}{30}$ *oz.* $\frac{2}{15}$ *Anſr.*

The Anſwer to this Queſtion wrought Decimally is .1333 (the 3 ſtill repeating) which wants of the complete Value found by the Rule for valuing the Remainder, $\frac{25}{7500}$ *oz.* or $\frac{1}{3000}$.

Exa.

Exa. 2. If $2\frac{4}{7}$ *lib.* coſt $3\frac{5}{9}$ *L.* what will $40\frac{5}{6}$ *lib.* coſt at the ſame Rate?

State $2\frac{4}{7} : 3\frac{5}{9} :: 40\frac{5}{6}$

By reducing the Fract. to Decimals. 2.571428 : 3.55555 :: 40.83333

40.83333

1066665
1066665
1066665
1066665
2844440
1422220

2.571428)145.1849464815(56.460825 *L.*

12857140 — 20

16613546 — 9.216500
15428568 — 12

11849784 — 2.598000
10285712 — 4

15640728 — 2.392000
15428568

21216015
20571424

6445910
5142856

13030540
12857140

173400

After reducing the common Fractions to Decimals, and multiplying and dividing according to the Rules, I continue the Quote to 2 Places further, by adding two 0's to the Remainders, and after the Quote is qualified,

fied, I find the Value of the Fractional Part: And here you are to obſerve, that all the Decimals being Circulates or incomplete; the Anſwer is not altogether exact, becauſe I have made no Allowance for the Increaſe that would have come out in the Product, if I had carried the Decimals of the laſt Terms to further Places before multiplying: However, the Defect is very inconſiderable, which will be manifeſt by comparing the Decimal with the common Way.

Exa. 3. If $2\frac{3}{4}$ *ſb.* buy 4 *oz.* of any thing, how much will 5 *L.* buy?

	ſb.	*oz.*	*l.*
State	$2\frac{3}{4}$	4	5
When reduced to	2.75	4	5
the Fract. of 1 *L.*	.1375	4	5

5

.1375)20.000(145.4545

1375

6250

5500

7500

6875

6250

5500

7500

6875

6250

5500

7500

6875

625

oz. 16

7.2720

oz. dr.

Anſr. 145—7 and ſomewhat more.

Here

Here I reduce 2 $\frac{3}{4}$ *ſh.* to the Decimal of 2 *L.* (becauſe the third Term is *L*) and after multiplying and dividing, I qualify the Quote, and it makes 145.4545, and there is ſtill a Remainder of 45, which would happen though I ſhould have continued the Diviſion *in infinitum*: However the Defect is ſo very inconſiderable, that the Quote wants only $\frac{4}{1375}$ of a Quarter of a Dram, as you may prove by common Fractions.

Exa. 4. A, B, C and D enter into Company: A put in 1375 *L.* B 180 *L.* C 975 *L.* and D 547 *L.* By trading they gain 463 *L.* How much of the ſame ought each Partner to receive in Proportion to his Stock?

A 1375
B 180
C 975
D 547

3077 Sum of the Stocks.

Then 1375×.15047123=206.89794125
=*L.* 206—17—11—2 for A's Share.
And 180×.15047123=27.0848214
=*L.* 27—1—8—1 for B's Share.
Alſo 975×.15047123=146.70944925
=*L.* 146—14—2—1 for C's Share.
Laſtly, 547×.15047123=82.30776281
=*L.* 82—6—1—3 for D's Share.

L. *L.* *L.*
3077 : 463 :: 1
1

3077) 463.00(.15047123, *&c.* Proportional Share of 1 *L.*
3077

15530
15385

14500

Brought over 14500
12308

21920
21539

3810
3077

7330
6154

11760
9231

2529

Share of { A 206—17—11—2 / B 27— 1— 8—1 / C 146—14— 2—1 / D 82— 6— 1—3 } Add

Here the Loſs is 1 *far.* 462—19—11—3 Proof.

Here I take the Sum of the Stocks, and with it and the Gain I find the Proportion Gain of 1 *L.* to be .15047123, *&c.* which multiplied by each's Stock gives their ſeveral Gains. The Reaſon why the Proof wants of the total Gain is, becauſe the Gain of 1 *L.* was incomplete. And ſo of others.

Exa. 5. What is the Value of 17 *C.* 3 *qrs.* 16 *lib.* of any Commodity, when 2 *C.* 1 *qr.* 17 *lib.* of the ſame coſt *L.* 6—19—6?

28 { 4 | 17.000000
7 | 4.250000
4 | *1.607143 *ferè*
.4017857 Decimal of 1 *qr.* 17 *lib.*

28 { 4 | 16.00000
   { 7 | 4.00000
   4 | *3.57143 *ferè*
.892857 Decimal of 3 *qrs.* 16 *lib.*

12 | 6.000
20 | 19.500
.975 Decimal of 19 *ſh.* 6 *d.*

C. *L.* *C.*
State 2.4017 : 6.975 :: 17.892
6.975

89460
125244
161028
107352

L.
2.4017)124796700(51.9618=*l.* 51-19-2-3

Here I reduce the mix'd Numbers of each Term to Decimals of the higheſt, by the expeditious Method mentioned before.

Exa. 6. If 6 *d.* gain 1 $\frac{1}{3}$ *far.* how much *per cent.* is gained at the ſame Rate?

ſh. *ſh.* *L.*
State .5 : .025 :: 100
100

.5)2.500(5 *l.* *Anſr.*
25

Here I reduce the 2 firſt Terms, each to the Decimal of 1 *ſh.* and the Anſwer comes out in *L.* becauſe the Proportion is, as *ſh.* are to *ſh.* ſo are *L.* to *L.*

This laſt Queſtion wrought by Reduction the common Way, will ſtand as below.

d. f. L.
6 : 1 $\frac{1}{5}$ :: 100
5 20

6 2000
12

24000

6 the middle Number.

First Term 6)144000
5)24000
4)4800 *far.*
12)1200 *d.*
2lb)10lb *sh.*
5 *L. Anſr.*

Or thus:

d. f. L.
6 : 1 $\frac{1}{5}$ :: 100
20

2000
12

24000
1 $\frac{1}{5}$

24000
4800

6)28800
4)4800
12)1200
2lb)10lb
5 *L.*

Or thus:

d. f. L.
6 : 1 $\frac{1}{5}$ :: 100
20

2000
12

24000

When reduced,

d. f.
6 : 1 $\frac{1}{5}$ :: 24000
1 : 1 $\frac{1}{5}$ :: 4000 by dividing by 6.
1 $\frac{1}{5}$

4000
800

4)4800
12)1200
2lb)10lb
5 *L.*

Or

Or thus:

$$\frac{6}{240} = \frac{1}{40} : \frac{6}{5} :: \frac{100}{1}$$

$$\frac{6}{5} \times \frac{100}{1} = \frac{600}{5} = 120$$

Then $\frac{1}{40})120(4800$ *far.* $= L.\ 5$, as by the other four Methods.

There are yet ſeveral other ways of working this ſame Queſtion; but I ſhall forbear inſerting them, having dwelt too long upon this Rule already. Take only one Queſtion more with the Anſwer, the Operation being left for your Exerciſe.

Exa. 7. A Gentleman has *L.* 100 *Sterling*, which with the Intereſt thereof (ſuppoſe at 5 *per cent.*) he is to ſpend in 20 Years, ſo as to ſpend equally each Year, and to exhauſt the whole Sum and Intereſt at the End of 20 Years: *Quær.* How much he muſt ſpend yearly?

Anſr. L. 80—4—10—1 *ferè per annum.*

CHAP. XXIII. *Rule of Five Numbers.*

IN this Rule are given five Numbers to find a ſixth, of which three contain a Suppoſition, and the other two a Demand.

All Queſtions in this Rule may be ſolved, either by two Operations of the Rule of Three Numbers; or, by one Diviſion. I ſhall begin with the firſt Method; for which take the following Directions.

1. Of the three Terms of the Suppoſition, ſet firſt down that one which is like the Thing ſought, the other two place on the left, (it matters not in what Order) and thoſe that belong to the Demand, ſet on the right, ſo as the fourth may be of the ſame Name with the firſt, and the fifth of the ſame Name with the ſecond.

2. Take the firſt, third and fourth Terms, and with

theſe make a Queſtion in the Rule of Three, and find an Anſwer.

3. Take the ſecond Term, the Anſwer to this firſt Queſtion, and the fifth Term, and make another Queſtion of the Rule of Three, the Anſwer to this laſt is the Anſwer to the general Queſtion, or the Thing required.

Exa. 1. If 8 Men are boarded 6 Months for *L.* 20, how much will ſerve 32 Men for 4 Months?

Men Mon. L. Men Mon.
State 8 : 6 : 20 :: 32 : 4

Men L. Men.
1*ſt*, 8 : 20 :: 32
20

8)640
80

By the firſt Operation I find, that if 8 Men ſpend *L.* 20 (in any Time) 32 Men (in the ſame Time) will ſpend *L.* 80.

2*dly*, *Mon. L. Mon.*
6 : 80 :: 4
4

6)320
L. 53—6—8

By the ſecond Operation, I find that if 8 Men (or any Number of Men) be maintained 6 Months for *L.* 80 the ſame Number will be maintained 4 Months for *L.* 53—6—8. Both theſe Proportions are direct.

Exa. 2. If 12 *C.* of any thing carried 100 Miles coſt *L.* 5—12, how many *C.* may be carried 150 Miles for *L.* 12—12?

L

L. ſh. M. C. L. ſh. M.

State 5—12 : 100 : 12 :: 12—12 : 150

1ſt, 5—12 : 12 : : 12—12

20 20

112 252

12

112)3024(27

224

784

784

M. C. M.

2dly, 100 : 27 : : 150

100

15|0)270|0

18 *C. Anſr.*

In the ſecond Operation I conſider, that if 27 *C.* be carried 100 Miles (for any Sum) leſs Weight muſt be carried 150 Miles for the ſame Sum: So that the firſt Proportion is direct and the other inverſe.

Exa. 3. What Principal Sum will gain *L.* 3—7—6 in 9 Months, when *L.* 100 gains *L.* 6 in 12 Months?

L. M. L. L. ſh. d. M.

State 6 : 12 : 100 :: 3—7—6 : 9

1ſt, 6 : 100 : : 3—7—6

20

67

12

810

100

6)81000

12)13500

2|0)1125

56—5

Firſt, I find that if *L.* 6 is gained by *L.* 100 (in any time) *L.* 3—7—6 will be gained by *L.* 56—5 in the ſame time.

2dly,

2dly, *M.* *L.* *ſh.* *M.*
12 : 56—5 : : 9
12

9)675—00
L. 75—00 *Anſr.*

In the ſecond Operation I conſider, that if 12 Months require *L.* 56-5 (or any other Principal) to gain any Sum, 9 Months will require a greater Principal to gain the ſame Sum, becauſe the time is ſhorter, therefore the Proportion is inverſe.

Exa. 4. If the Produce of 6 Pecks of Corn be 4 Bolls 14 Pecks, 1 Lippy in a Year, what would 19 Bolls 8 Pecks yield in 7 Years at the ſame Rate (ſuppoſing that 19 Bolls 8 Pecks be ſown each of theſe 7 Years?)

P. *Y.* *B.* *P.* *lip.* *B.* *P.* *Y.*
State 6 : 1 : 4—14—1 : : 19—8 : 7

1ſt, 6 : 4—14—1 : : 19—8
16 16
78 122
4 19
313 312
312
626
313
939
6)97656
16276 *lip.*

Y. *lip.* *Y.*
2dly, 1 : 16276 : : 7
7
4)113932
16 { 4 | 28483 Pecks.
4 7120 3
1780—3

B. *Pecks.*
Anſr. 1780—3

Exa. 5. If 6 *L.* is the Intereſt of 100 *L.* for 12 Months, what is the Intereſt of 75 *L.* for 9 Months?

L. *M.* *L.* *L.* *M.*
State 100 : 12 : 6 : : 75 : 9

M.

M. L. ſh. M.

1*ſt*, 100 : 6 :: 75 2*dly*, 12 : 4—10 :: 9

6 9

4|50 12)40—10

20

10|00 *L.* 3— 7—6 *Anſr.*

Exa. 6. If 75 *L.* in 9 Months gain *L.* 3—7—6, what is the Intereſt of *L.* 100 for 12 Months?

State 75 : 9 : 3—7—6 :: 100 : 12

L. d. L. M. d. Mon.

1*ſt*, 75 : 810 :: 100 2*dly*, 9 : 1080 :: 12

100 12

75)81000(1080 *d.* 9)12960

12)1440

2|0)12|0

6 *L. Anſr.*

Exa. 7. What Principal Sum will raiſe *L.* 6 in 12 Months, when 75 *L.* raiſes *L.* 3—7—6 in 9 Months?

L. ſh. d. M. L. L. M.

State 3—7—6 : 9 : 75 :: 6 : 12

M. L. M.

1*ſt*, 3—7—6 : 75 : : 6 2*dly*, 9 : 133$\frac{1}{3}$ :: 12

20 20 9

67 120 12)1200

12 12 100 *L. Anſr.*

810 1440

75

7200

10080

81|0 $\left\{\begin{matrix}9\\9\end{matrix}\right.$ 1080|0

1200

L. 133$\frac{1}{3}$ *Exa.*

Exa. 8. What Time will *L.* 75 take to gain *L.* 3—7—6, when *L.* 100 gains *L.* 6 in 12 Months?

State 100 : 6 : : 12 : : 75 : 3—7—6

1st, 100 : 12 : : 75

100

75)1200(16
75

450
450

2dly, 6 : 16 : : 3—7—6

20 20

120 67
12 12

1440 810
16

4860
810

144)12960(90
12 108

9 *Mon. Anſr.*

Exa. 9. If 75 *l.* in 9 Months gain *l.* 3—7—6, what Time will *l.* 100 take to gain *l.* 6?

State 75 : 3—7—6 : 9 : : 100 : 6

L. ſh. d. M. L.

1st, 75 : 9 : : 100

9

100,675(6¾

2dly, 3—7—6 : 6¾ : : 6

20 20

67 120
12 12

810 1440
6¾

8640
360
720

81)9720(108
9

12 *Mon. Anſr.*

And thus I have varied the third Example, and turn'd it into all its Shapes, by continually altering the Demand.

Exa. 10. How many Buſhels of Corn will 150 Horſes eat up in 40 Days at the Rate of 14 Buſhels for three Horſes in 5 Days?

H. D. B. H. D.
State 3 : 5 : 14 :: 150 : 40

H. B. H.
1*ſt*, 3 : 14 :: 150
14

3)2100
700

By the firſt Operation I find, that if 3 Horſes eat up 14 Buſhels in any Time, 150 Horſes in the ſame Time will eat up 700 Buſhels.

D. B. D.
2*dly*, 5 : 700 :: 40
40

5)28000
5600

Then, if 5 Days ſpend 700 Buſhels, 40 Days will ſpend 5600 Buſhels.

In the Operations of theſe 10 Queſtions, I have uſed none of the contracted Methods mentioned in *Contractions in the Rule of Three*, particularly in *Caſe* V. that the Learner may the better underſtand the Rule; though I might have conſiderably abbreviated the moſt of them.

When there is any Remainder in the firſt Operation, let it be reduced to the loweſt Denomination; and if any Thing yet remains, annex it fractionwiſe to the Number of the loweſt Species, and make it a Part of the middle Number in the ſecond Operation: For if you ſhould neglect the firſt Remainder, or even the laſt, the final Anſwer would be incomplete, and in many Caſes the Defect very conſiderable.

Now I proceed to ſhew how all Queſtions in this Rule may be ſolved by one Operation, which Method is much eaſier, more exact, and more compendious than the preceding.

RULE.

RULE.

State the five Numbers as before, and reduce the correſponding Terms, *viz.* 1ſt and 4th, alſo 2d and 5th to the ſame Denomination ſeverally (if neceſſary) and the middle Number (if mix'd) to a ſimple one.

2. Compare the firſt, third and fourth Terms, and find what Extreme would be Diviſor, if working by two Operations; which mark: Then compare the ſecond, third and fifth, marking the Extreme that would be Diviſor there.

3. Which two Terms thus marked, multiply together for a Diviſor, and the other three for a Dividend, the Quote reſulting from this Diviſion is the Anſwer to the Queſtion, in the ſame Name with the middle Number.

For Examples, we ſhall repeat thoſe adduced for exemplifying the Rule of Five Numbers by two Operations, to let you ſee the Conſonancy of both Methods: But be ſure to have a clear Underſtanding of the firſt Method before you commence this.

Exa. 1. repeated.

Men	*Mon.*	*L.*	*Men*	*Mon.*
×8	: ×6	: 20	: 32	: 4
6			20	
48			640	
			4	

48 { 6)2560
   { 8) 426—13—4
L. 53— 6—8

Here I ſay, If 8 Men ſpend *L.* 20 (in any Time) 32 Men will ſpend more in the ſame Time; therefore I divide by the leſſer Extreme, *viz.* 8, which I mark thus ×. Then I ſay, If 6 Months ſpend 20 *l.* (or

(or any Sum) 4 Months will ſpend leſs; therefore, I divide by the greater Extreme; *viz.* 6; which two ſo marked, I multiply together for a Diviſor, and the other three for a Dividend, and the Anſwer is *L.* 53-6-8, as before.

Exa. 2. repeated.

L. *ſh.* *M.* *C.* *L.* *ſh.* *M.*

5—12 : 100 : 12 :: 12—12 : X 150

20 20

X 112 252

150 100

560 25200

112 12

168|00)3024|00(18 C.

168

1344

1344

Here I reduce the firſt and fourth Terms to *ſh.* and finding that if 112 *ſh.* carry 12 *C.* (any length) 252 *ſh.* will carry more weight the ſame length, I mark the leſſer Extreme; 112 for one of the Diviſors; then if 12 *C.* be carried 100 Miles for any Sum of Money, I conſider that leſs Weight will be carried 150 Miles for the ſame Sum; therefore I mark the greater Extreme; *viz.* 150 for the other Diviſor, and after multiplying and dividing according to the Rule, the Quote is 18 *C.* for the Anſwer.

Exa. 3. repeated.

6 : 12 : 100 : : 3—7—6 : × 9

20	20
120	67
12	12
× 1440	810
: 9	100
12960	81000
	12

12960)9720010(75 *L. Anſr.* as before.

9072

6480

6480

Exa. 4 repeated.

P. Y. B. *p. lip.* B. *p.* Y.

× 6 : × 1 : 4—14—1 : : 19—8 : 7

16	16
78	122
4	19
313	312
	313
	936
	312
	936
	97656
	7
	683592

Brought

Brought over 6|583592
4|113932
16 } 4 | 28483
4 | 71203
1780—3 *Anſr.* as before.

Exa. 5. repeated.

L.	*M.*	*L.*	*L.*	*M.*
× 100 :	× 12 :	6 : :	75 :	9
12			9	
1200			675	
			6	
			4050	

120|0 { 10 | 40|0
12 | 40—10
L. 3— 7—6 *Anſr.* as before.

Exa. 6. repeated.

L.	*M.*	*L. ſh. d.*	*L.*	*M.*
× 75 :	× 9 :	3—7—6 : :	100 :	12
9		20		
675		67		
		12		
		810		
		100		
		81000		
		12 12		

675)972000 (1440 *d.*
675 · · · 2|0)12|0 *ſh.*
6 *L. Anſr.* as before.
2970
2700
2700
2700

Exa. 7. repeated.

3—7—6 : 9 : 75 :: 6 : 12 ×

20	20
67	120
12	12
× 810	1440
12	75
9720	7200
	10080
	108000
	9

9720)972000(100 *l. Anſr.* as before.

Exa. 8. repeated.

L. L. M. L. L. ſb. d.

100 : 6 : 12 :: × 75 : 3—7—6

20	20
120	67
12	12
× 1440	810
75	12
7200	9720
10080	100
108000	972000(9 M. *Anſr.* as before.
	972

Exa.

Exa. 9. repeated.

75 : 3—7—6 : 9 :: × 100 : 6

20	20
67	120
12	12
× 810	1440
100	9
81000	12960
	75
	64800
	90720

81 } 9 | 972000
} 9 | 108

12 Mon. *Anſr.* as before.

Exa. 10. repeated.

H. D. B. H. D.

× 3 : × 5 : 14 :: 150 : 40

5	40
15	6000
	14

15 } 3 | 84000
} 5 | 28000

5600 Buſh. *Anſr.* as before.

Exa. 11. If 12 Men build a Wall 30 Foot long, 6 Foot high, and 3 Foot thick, in 15 Days, in how many Days will 60 Men build 300 Foot long, 8 Foot high and 6 Foot thick (according to the ſame Rate?) 30x6x3=540 ſolid Feet, and 300x8x6=14400 ſolid Feet.

M.	*Feet.*	*D.*	*M.*	*Feet.*
12 :	x 540 :	15 : :	x 60 :	14400
	60			15
Diviſor	324|00			216000
				12

324)25920|00(80 Days, *Anſ.*
2592
0

APPENDIX.

APPENDIX.

CHAP I.

I. *Of Simple Intereſt.*

ALL Queſtions relating to ſimple Intereſt may be ſolved by the Rule of Three: Numbers, or that of Five: Numbers, after the Method of *Exa.* 11, 16, 22, 25, 31, Pages, 147, 148, 152, 155, and 160; or of *Exa.* 3, 5, 6, 7, 8, 9, Pages, 199, 200, 201 and 202.

Or, by finding the Intereſt of 1 *L.* for 1 Day, at the Rate *per cent.* propoſed, and thereby multiplying the Sum whoſe Intereſt is required, and the Product by the Number of Days for the Anſwer.

Now, the Int. of 1 *L.* for 1 Day at
5 *per cent.* is - - .00013698, *&c.*
Alſo, the Int. of 1 *L.* for 1 Day at
6 *per cent.* is - - .00016438, *&c.*

Found thus:

L. L. L. *D. L. D.*
100 : 5 :: 1 : .05. Then, 365 : .05 :: 1 : .00013698, *&c.*

Alſo,

L. L. L. *D. L. D.*
100 : 6 :: 1 : .06. Then, 365 : .06 :: 1 : .00016438, *&c.*

And ſo for any other Rate *per cent.*

Exa. 25, Page 175. repeated and wrought this Way.

.00016438
546

00098628 *Anſr.* L. 76—0—4½

00065752
00082190

00.08975148
847

62826036
35900592
71801184

76.01950356 Intereſt.

Another Example. What is the Intereſt of *L.* 420 for 160 Days at *L.* 5 *per cent. per annum*?

.00013698
420

27396
54792

.05753160
160

L. 9.205056 *Anſr.* or *L.* 9—4—1¼ *ferè* by this Method.

The ſame Example wrought by the Rule of Five.

100 : 365 : 5 :: 420 : 160
160

67200
5

365|00)336|0|00(9 *L.*
3285

Brought

Brought over 3285

75
20

365.00)1500(4 *ſh.*
1460

40
12

)480(1 *d.*
365

115
4

460(1 *f.*
365

95

By this Method the Anſwer is more complete.

II. *Of Rebate or Diſcount at Simple Intereſt.*

This you have in *Exa.* 35. Page 162, which it is unneceſſary to repeat. I ſhall therefore propoſe another Method; namely, Find the preſent Worth of 1 *L.* due at the end of any Number of Years to come, diſcounting at a propoſed Rate *per cent.* by which multiplying the Sum propoſed, the Product is the preſent Worth required.

Now, to find the preſent Worth of 1 *L.* due at the end of any Number of Years to come, diſcounting ſimple Intereſt at any Rate *per cent.* uſe the following Method.

If the Intereſt is at 5 *per cent.* ſay,

Years.	*L.*	*L.*	*L.*	*L.*
1	105	100	1	.952381+
2	110	100	1	.909091+
3	115	100	1	.869565—
4	120	100	1	.833333—

And ſo on to as many Years as you pleaſe.

But if the Diſcount be computed at *L.* 6 *per cent. per annum*, the Proportions will be,

Years.	*L.*	*L.*	*L.*	*L.*
1	106	100	1	.943396
2	112	100	1	.892857
3	118	100	1	.847457
4	124	100	1	.806451

And ſo for any other Rate *per cent.* Thus are calculated the Numbers in the following Table I. extending to 30 Years, which may be continued to any Number of Years you pleaſe. And tho' I have only made one at 5 *per cent.* you may by the ſame Method (*mutatis mutandis*) compoſe other Tables at other Rates.

TABLE

TABLE I.

Shewing in Decimal Parts of 1 *L.* the preſent worth of 1 *L.* due at the End of any Number of Years to come under 31, diſcounting at the Rate of 5 *per cent. per annum*, Simple Intereſt.

Years.		*Years.*	
1	.952381+	16	.555555—
2	.909091+	17	.540540—
3	.869565+	18	.526316+
4	.833333—	19	.512820—
5	.8	20	.5
6	.769231+	21	.487804—
7	.740740—	22	.476191+
8	.714286+	23	.465114—
9	.689655—	24	.454545—
10	666666—	25	.4
11	.645161—	26	.434783+
12	.625	27	.425532+
13	.606060—	28	.416666—
14	.588235—	29	.408163—
15	.571428—	30	.4

Such as are marked with the Sign + are ſomewhat more than the Truth, and thoſe marked with the Sign — are ſomewhat leſs.

Exa. What is the preſent Worth of *L.* 150 due at the end of 5 Years, Rebate being computed at 5 *L. per cent. per annum*, Simple Intereſt?

Tabular Number againſt 5 Years is -	.8
Which multiplied by - - -	150
Gives *L.* 120 for the Anſwer -	120.0

Operation

Operation after the common Way.

125 : 100 : : 150
150

125)15000(120
125

250
250

o And ſo of others.

III. *Of Annuities in Arrears at Simple Intereſt.*

When an Annuity is in Arrear for any Number of Years, and you want to know the Amount of it, Simple Intereſt being computed for each particular Payment from the time it became due to the End of the Number of Years propoſed, work as in the following Example.

There is an Annuity of *L.* 150 forborn to the End of 5 Years, what is then due, Simple Intereſt being computed at 5 *per cent?*

Here you muſt find the Intereſt of *L.* 150 for 4 Years, 3 Years, 2 Years and 1 Year: All which being added to the Sum of the yearly Annuity, the Aggregate is the Anſwer or Amount of *L.* 150 Annuity foreborn for 5 Years at Simple Intereſt.

Now, the Intereſt of *L.* 150 at 5 *per cent.* for	1	Years is	7 : 10
	2		15 :
	3		22 : 10
	4		30 :
Sum of the Intereſts,			75 :
5 times 150, *viz.* the Sum of the Annuity,			750 :
Amount, *L.*			825 :

But

But the Intereſt of any Sum for 4 Years + 3 Years + 2 Years + 1 Year, is equal to the Intereſt of that Sum for 10 Years; wherefore, to find the Sum of the Intereſt at once, take the natural Series of Numbers, 1, 2, 3, *&c.* to the Number of Years leſs 1, and find the Intereſt for the Sum hereof, which added to the Sum of the yearly Annuity gives the Amount as before.

Thus in the laſt *Exa.* 1+2+3+4=10 Years.

	L.	*Y.*	*L.*	*L.*	*Y.*
And	100	: 1 :	5	: : 150	: 10

10

1500

5

Intereſt, *L.* 75|00

Sum of the Annuity, 750

L. 825 *Anſr.*

Hereupon is grounded the Conſtruction of the following

TABLE II.

Shewing in *L.* and Decimal Parts of a *L.* the Amount of 1 *L.* Annuity being forbore to the end of any Number of Years under 31, Simple Intereſt being computed at 5 *per cent. per annum.*

1	1.	11	13.75	21	31.5
2	2.05	12	15.3	22	33.55
3	3.15	13	16.9	23	35.65
4	4.3	14	18.55	24	37.8
5	5.5	15	20.25	25	40.
6	6.75	16	22.	26	42.25
7	8.05	17	23.8	27	44.55
8	9.4	18	25.65	28	46.9
9	10.8	19	27.55	29	49.3
10	12.25	20	29.5	30	51.75

Exa. There is an Annuity of *L.* 150 forborn to the End of 5 Years, what is then due, Simple Intereſt being computed at 5 *per cent. per annum?*

Tabular Number againſt 5 Years,	5.5
The Annuity, - -	150
Anſr. *L.* 825.0	

IV. *Of the preſent Worth of Annuities at Simple Intereſt.*

When an Annuity is to be ſold for ready Money at Simple Intereſt, its Value will be found by the Method of operating the following Queſtion.

There is an Annuity of *L.* 150 to continue 5 Years to come, what is it worth in ready Money, Rebate being allowed at 5 *per cent.* Simple Intereſt?

Here you muſt find the preſent Worth of *L.* 150 due at the End of 1 Year, the preſent Worth of *L.* 150 due at the End of 2 Years, its preſent Worth due at the End of the 3d Year, likewiſe its preſent Worth due at the End of the 4th and 5th Years, the Sum of all which is the preſent Worth of the Annuity.

Theſe ſeveral Worths are thus found:

Year.					
1	105 :			150 :	142.85714285
2	110 :			150 :	136.36363636
3	115 :	}	100 : :	150 :	130.43478261
4	120 :			150 :	125.
5	125 :			150 :	120.

L. 654.65556182 *Anſr.*

or *L.* 654 : 13 : 1$\frac{1}{4}$ and a Fraction more.

Whence

Whence is construed the following TABLE III. shewing the present Worth of *L.* 1 Annuity to continue any Number of Years under 31, Simple Interest being computed at 5 *per cent.* Thus:

105 : 100 :: 1 :	.952381	{	Pres. Worth of 1 *l.* due at 1 year's end.
110 : 100 :: 1 :	.909090	{	Pres. Worth of 1 *l.* due at 2 years end.
	1.861471	{	Pres. W. of 1 *l.* Ann. due at 2 years end.
115 : 100 :: 1 :	.869565	{	Pres. Worth of 1 *l.* due at 3 years end.
	2.731036	{	Pres. W. of 1. *l.* Ann. due at 3 years end.
120 : 100 :: 1 :	.833333	{	Pres. Worth of 1 *l.* due at 4 years end.
	3.564369	{	Pres. W. of 1 *l.* Ann. due at 4 years end.

TABLE III.

Shewing the present worth of 1 *L.* Annuity to continue any Number of Years under 31, Simple Interest being computed at 5 *per cent.*

Years.		*Years.*	
1	.952381	16	11.536386
2	1.861471	17	12.076926
3	2.731036	18	12.603242
4	3.564369	19	13.116062
5	4.364369	20	13.616062
6	5.133600	21	14.103866
7	5.874340	22	14.580057
8	6.588626	23	15.045171
9	7.278281	24	15.499716
10	7.944947	25	15.944160
11	8.590108	26	16.378943
12	9.215108	27	16.804475
13	9.821168	28	17.221141
14	10.409403	29	17.629304
15	10.980831	30	18.029304

Exa. What present Money will satisfy for an Annuity of *L.* 150 to continue 5 Years, Rebate being made at 5 *per cent. per annum*, Simple Interest?

Tabular Number for 5 Years, 4.364369

150

21821845
4364369

Anſr. L. 654.65535

And now, for the Solution of all Questions concerning Annuities in Arrears at Simple Interest, take the following Cases.

CASE

CASE I.

Having the Annuity, Time and Rate (*viz.* the Intereſt of 1 *L.* for a Year) to find the Amount.

RULE. Take the natural Series of Numbers, 1, 2, 3, *&c.* to the Number of Years leſs 1, the Sum of this Series multiply by one Year's Intereſt of the Annuity, and the Product is the whole Intereſt due upon the Annuity, to which adding the Total of the Annuities, the Sum is the Amount required.

Exa. What is the Amount of *L.* 150 Annuity for 5 Years, allowing Simple Intereſt for each Year after it falls due at 5 *per cent?*

1+2+3+4=10. Then .05×150=7.5 and 7.5×10 =75 and 150×5=750. Laſtly, 750+75=825, the Amount required.

CASE II.

Having the Amount, Rate and Time, to find the Annuity.

RULE. Take the Sum of the natural Series of Numbers, 1, 2, 3, *&c.* as before, to the Number of Years leſs 1, which multiply by the Rate, and to the Product adding the Years by this Sum, divide the Amount, and the Quote is the Annuity.

Exa. What Annuity will in 5 Years amount to *L.* 825 at 5 *per cent.* Simple Intereſt?

Thus 1+2+3+4=10. Then 10×.05=.5 and .5+5 =5.5. Laſtly, 5.5)825(150 *Anſr.*

CASE. III.

Having the Annuity, Amount and Time, to find the Rate.

RULE. Take the Difference betwixt the Amount and the Product of the Annuity and Time, which di-

vide by the Product of the Annuity, multiplied into the Sum of this Series, 1, 2, 3, &c. to the Number of Years leſs 1, and the Quote is the Rate.

Exa. At what Rate of Intereſt will an Annuity of *L.* 150 amount to *L.* 825 in 5 Years, Simple Intereſt?

150×5=750; then 825—750=75 for a Dividend, and 150×10=1500 for a Diviſor.

1500)75.00(.05 *Anſr.*

CASE IV.

Having the Annuity, Amount and Rate, to find the Time.

RULE. Divide twice the Amount by the Product of the Rate into the Annuity, then ſubtracting the Product of the Rate into the Annuity, from double the Annuity, divide the Difference by the Product of the Rate into the Annuity, and ſquare the Quote, dividing the Product by 4; which laſt Quote add to the firſt, and from the Square Root of the Sum ſubtract ½ of the Number you ſquared, and the Remainder is the Anſwer or Time ſought.

Exa. What time will *L.* 150 Annuity take to amount to *L.* 825 at 5 *per cent. per annum*, Simple Intereſt?

825 2 = 1650 twice the Amount.

150 .05 = 7.50 Product of the Rate into the Annuity.

7.5)1650(220

150

150
2

300
7.5 Subtr.

7.5)292.5(39

Then 39×39=1521 and

4)1521

380.25 } Add
220. }

600.25(24.5
4

44)200
176

485)2425
2425

Laſtly, $\frac{19}{2}$ = 19.5 and 24.5—19.5=5 *Anſr.*

Of the Purchaſe of Annuities at Simple Intereſt.

CASE V.

Having the Annuity, Rate and Time, to find the preſent Worth.

For the Solution of this Caſe, work as in Article 4th, Page 217.

CASE VI.

Having the preſent Worth, Rate and Time, to find the Annuity.

RULE. Take any Annuity at Pleaſure, and find its preſent Worth as before; then ſay, As that preſent Worth is to its Annuity, ſo is the given preſent Worth to its Annuity.

Exa. What Annuity to continue 5 Years is worth 654.65535 preſent Money, allowing Simple Intereſt at 5 *per cent?*

The

The preſent Worth of 1 *L.* Annuity for 5 Years at 5 *per cent.* is 4.364369; therefore,

4.364369 : 1 :: 654.65535 : 150 *Anſr.*

CASE VII.

Having the Annuity, preſent Worth and Rate, to find the Time.

RULE. Divide the given Annuity by the Series of the Amounts of 1 *L.* Annuity for 1, 2, 3, *&c.* Years, taking the Sum of the Quotes at every Step, and thus proceed till you find a Sum equal to the given preſent Worth; and the Number of Diviſions is the Number of Years required.

Exa. What Time muſt an Annuity of 150 *L.* continue, to be worth 654.6555617, ready Money, at the Rate of 5 *per cent.* Simple Intereſt?

1.05)150(142.8571428=1 Year.
1.1)150(136.3636363=2 Years.

279.2207791
1.15)150(130.4347826=3 Years.

409.6555617
1. 2)150(125. =4 Years.

534.6555617
1.25)150(120. =5 Years.

654.6555617 *Anſr.* 5 Years.

CHAP.

CHAP. II.

Of Compound Intereſt.

WHEN the Amount of any Sum at Compound Intereſt is required, work as in Queſt. 39. Page 165, only it is eaſier to do it by Decimal Fractions.

But all ſuch Queſtions are more expeditiouſly ſolved by finding the Amount of 1 *L.* for any Number of Years at any Rate *per cent.* by which multiplying the Sum propoſed, the Product is the Anſwer.

Thus, Intereſt being at 5 *per cent.*

Years.	
1	100 : 105 : : 1 : 1.05
2	100 : 105 : : 1. 05 : .1.1025
3	100 : 105 : : 1.1025 : : 1.157625
4	100 : 105 : : 1.157625 : : 1.21550625

And thus is conſtrued the following

TABLE

TABLE IV.

Shewing the Amount of 1 *L.* forborn to the End of any Number of Years under 31, Compound Intereſt being computed at 5 *per cent.*

Years.		*Years.*	
1	1.05	16	2.1828746
2	1.1025	17	2.2920183
3	1.157625	18	2.4066192
4	1.2155063	19	2.5269502
5	1.2762816	20	2.6532977
6	1.3400956	21	2.7859626
7	1.4071006	22	2.9252607
8	1.4774554	23	3.0715238
9	1.5513282	24	3.225099
10	1.6288946	25	3.3863549
11	1.7103393	26	3.5556727
12	1.7958563	27	3.7334563
13	1.8856491	28	3.9201289
14	1.9799316	29	4.1161356
15	2.0789282	30	4.3219424

Exa. What will *L.* 315: 10 amount to, being forborn 3 Years, Compound Intereſt being computed at 5 *per cent. per annum*?

Tabular Number for three Years, 1.157625

315.5

5788125
5788125
1157625
3472875

365.2306875

Anſr. L. 365: 4: $7\frac{1}{4}$

II. Of

II. *Of Rebate or Diſcount at Compound Intereſt.*

Exa. There is 365.2306875 due at the End of 3 Years to come, I demand how much preſent Money will ſatisfy for the ſaid Debt, diſcounting at the Rate of 5 *per cent. per annum*, Compound Intereſt?

L.	*L.*	*L.*	*L.*
105 :	100 : :	365.2306875 :	347.83875
105 :	100 : :	347.83875 :	331.275
105 :	100 : :	331.275 :	315.5 *Anſr.* or *L.* 315 : 10

And upon this is founded the Calculation of the following Table: For,

L.	*L.*	*L.*	*L.*
105 :	100 : :	1.952381 *ferè.*	
105 :	100 : :	.952381 :	.907029, *&c.*
105 :	100 : :	.907029 :	.863837, *&c.*

TABLE V.

Shewing the preſent Worth of 1 *L.* due at the end of any Number of Years to come under 31, diſcounting at 5 *per cent. per annum*, Compound Intereſt.

1	.952381	16	.4581115
2	.9070294	17	.4362967
3	.8638376	18	.4155207
4	.8227025	19	.3957339
5	.7835262	20	.3768895
6	.7462154	21	.3589424
7	.7106813	22	.3418499
8	.6768394	23	.3255713
9	.6446089	24	.3100679
10	.6139133	25	.2953028
11	.584679	26	.2812407
12	.5568374	27	.2678483
13	.5303213	28	.2550936
14	.5050679	29	.2429463
15	.4810171	30	.2313774

Exa.

Exa. What preſent Money will ſatisfy for a Debt of 365.2306875 due at the end of 3 Years to come, diſcounting at 5 *per cent.* Compound Intereſt?

365.2306875
.863837

25566148125
10956920625
29218455000
10956920625
21913841250
29218455000

315.49978139793 75 *Anſr.* L. 315 : 10 *ferè*, becauſe the Tabular Number is ſomewhat too ſmall.

III. *Of the Amount of Annuities at Compound Intereſt.*

There is an Annuity of L. 150 forborn to the end of 5 Years, what is then due, Compound Intereſt being computed at 5 *per cent. per annum?*

L.	L.	L.	L.
100	105	150	157.5
100	105	157.5	165.375
100	105	165.375	173.64375
100	105	173.64375	182.3269375

150.

Here you muſt work as if it was a Principal lying out for any Number of Years at Compound Intereſt. For,

150

150 *L.* due at the end of the 5th Year is 150.
And 150 *L.* due at the end of the 4th Year,
will at the end of the 5th Year amount to 157.5
And 150 *L.* due at the end of the 3d Year,
will at the 5th Year's end amount to 165.375
And 150 *L.* due at the end of the 2d Year,
will at the 5th Year's end amount to 173.64375
And 150 *L.* due at the 1ſt Year's end, will
at the 5th Year's end amount to 182.326937

The Sum of all which is what is due at the 5th Year's end. } 828.845687

But ſuch Queſtions are more eaſily ſolved by the following

TABLE VI.

Shewing the Amount of 1 *L.* Annuity to continue any Number of Years under 31, Compound Intereſt being computed at 5 *per cent. per annum.*

Years.		*Years.*	
1	1.	16	23.6574918
2	2.05	17	25.8403664
3	3.1525	18	28.1323799
4	4.310125	19	30.5389989
5	5.5256312	20	33.0659489
6	6.8019128	21	35.7192518
7	8.1420079	22	38.5052144
8	9.5491079	23	41.4304679
9	11.0265639	24	44.501999
10	12.5778925	25	47.7270899
11	14.2067871	26	51.1134537
12	15.9171265	27	54.6691264
13	17.7129799	28	58.4025827
14	19.5986289	29	62.3226999
15	21.5785599	30	66.4388474

For 1 *L.* Annuity due at the end of 1 Year is 1.000000
And 1 *L.* Ann. due at the end of 2 Years is 2.05
Thus found;
100 : 105 :: 1 : 1.05, to which adding the 1 *L.* Annuity, the Sum is 2.05.
Alſo, 1 *L.* Ann. due at the end of 3 Years is 3.1525
Thus found;
100 : 105 :: 2.05 : 2.1525, to which adding the 1 *L.* Annuity the Sum is 3.1525.
And 1 *L.* Annuity due at the end of 4 Years is 4.310125
Thus found;
100 : 105 :: 3.1525 : 3.310125, to which adding the 1 *L.* Ann. the Sum is 4.310125.

And after the ſame Manner is the preceding Table calculated, where the Numbers are ſomewhat deficient of their true Value; ſo that the Anſwer found thereby, may perhaps want 1 Farthing, and in ſome Caſes 1 Halfpenny.

Exa. What is the Amount of *L.* 150 Annuity forborn 5 Years, at 5 *per cent. per annum*, Compound Intereſt?

5.5256312
150
27628156 00
55256312
Anſr. L. 828.8446800

IV. *Of the preſent Worth of Annuities at Compound Intereſt.*

The preſent Worth of Annuities at Compound Intereſt, is found after the Method of the following Example.

Exa. What is the preſent Worth of an Annuity of 150 *L.* to continue 5 Years, rebating at 5 *per cent.* Compound Intereſt?

		L.		L.	
L.	L.	150.	:	142.85714285	=1. year.
105	100 ::	142.85714285	:	136.05613605	=2. year.
		136.05613605	:	129.57727243	=3. year.
		129.57727243	:	123.40692612	=4. year.
		123.40692612	:	117.53040583	=5. year.

Sum of the ſaid preſ. Worths 649.42788328 *Anſr.*

Hence is, caculated the following

TABLE VII.

Shewing the preſent worth of 1 *L.* Annuity payable by yearly Payments, and to continue any Number of Years not exceeding 30, diſcounting at 5 *per cent.* Compound Intereſt

Years.		*Years.*	
1	.952381	16	10.8377695
2	1,8594103	17	11.2740662
3	2.7232479	18	11.6895869
4	3.5459499	19	12.0853208
5	4.3294759	20	12.4622103
6	5.0756999	21	12.8211527
7	5.7863734	22	13.1630026
8	6.6463212	23	13.4885739
9	7.1078217	24	13.7986418
10	7.7217349	25	14.0939445
11	8.3064142	26	14.3751853
12	8.8632516	27	14.6430336
13	9.3935689	28	14.8981272
14	9.8986408	29	15.1410735
15	10.3796581	30	15.3724509

	L.	L.	L.	L.
For	105	100	1	.952381=1ſt Year.
105	100	.952381		.907029
				1.859410=2d Year.
105	100	.907029		.863837
				2.723247=3d Year.
105	100	.863837		.822702
				3.545949=4th Year, &c.

Exa. What is the preſent Worth of an Annuity of *L.* 150 to continue 5 Years, Rebate being at 5 *per cent.* Compound Intereſt.

Tabular Number for five Years, 4.329475

150

21647375
4329475

649.421250 *Anſr.*

Here follow the Solutions of the moſt common Caſes concerning Annuities in Arrear at Compound Intereſt.

CASE I.

Having the Annuity, Rate and Time, to find the Amount.

RULE. Multiply the Annuity by the Amount of 1 *L.* for the Time, and at the Rate propoſed, and the Product is the Anſwer.

CASE II.

Having the Amount, Rate and Time, to find the Annuity.

RULE. Divide the Amount given by that of 1 *L.*

An-

Annuity for the Time, and at the Rate propoſed, and the Quote is the Anſwer.

Exa. What Annuity will amount to *L.* 828.84465 in 5 Years at 5 *per cent. per annum*, Compound Intereſt?

5.525631)828.84465(156 *Anſr.*

CASE III.

Having the Annuity, Rate and Amount, to find the Time.

RULE. Find a Principal of which 1 Year's Intereſt is equal to the given Annuity, the Sum of this and the given Amount is the Amount of that Principal for the given Rate and Time ſought: Then divide that Amount by its Principal, and multiply the Rate continually by itſelf till the Product be equal to the former Quote, and the Number of Multiplications is the Anſwer or Time ſought.

Exa. In what Time will *L.* 150 Annuity amount to 828.84465 at 5 *per cent.* Compound Intereſt?

.05 : 1 :: 150 : 3000, and 3000 + 828, *&c.* = 3828.84465

Then 3000)3828.84465(1.27628155 = 1.05 raiſed to the 5th Power, or multiplied 5 times into itſelf. So the Anſwer is 5 Years.

Of the Purchaſe of Annuities at Compound Intereſt.

CASE IV.

Having the Annuity, Rate and Time, to find the preſent Worth.

RULE. Find the preſent Worth of each Year's Annuity by itſelf, the Sum of all which is the preſent Worth ſought. Or,

2. Multiply the Annuity by the preſent Worth of 1 *L.* Annuity for the Time, and at the Rate propoſed, and the Product is the Anſwer.

CASE V.

Having the preſent Worth, Rate and Time, to find the Annuity.

RULE. Divide the preſent Worth given by that of 1 *L.* Annuity, and the Quote is the Anſwer.

Exa. What Annuity, to continue 5 Years, will 649.42125 purchaſe, allowing Compound Intereſt at 5 *per cent*?

4.329475)649.42125(150 *Anſr.*

CASE VI.

Having the Annuity, preſent Worth and Rate, to find the Time.

RULE. Find a Principal whereof 1 Year's Intereſt is the Annuity given, from which ſubtract the preſent Worth, and the Remainder is the preſent Worth of that Principal, conſidered as a Sum due at the End of the Annuity; then find what Time this preſent Worth will take, to amount to the Principal found.

Exa. What time muſt an Annuity of *L.* 150 continue to be worth 649 42125 in ready Money?

.05 : 1 :: 150 : 3000; then 3000—649.42125 = 2350.57875 preſent Worth of *L.* 3000, due at the end of the Time ſought.

Then 2350.57875)3000.00000(1.276281, *&c.* which being the 5th Power of 1.05,5; is the Number of Years ſought.

V. *To find the preſent Value of a Free-hold Eſtate or Annuity to continue for ever.*

RULE. Find a Principal of which 1 Year's Intereſt is the Rent or Annuity given, and this is the Price ſought.

Exa. What is the Price or preſent Value of an Annuity of 150 *L.* to continue for ever, diſcounting at 5 *per cent.* Compound Intereſt?

.05 : 1 :: 150 : 3000 *Anſr.*

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OF MENSURATION OF SUPERFICIES and SOLIDS.

SECT. I. *Of Superficies.*

PROP. I.

HAVING the Diameter of a Circle, to find the Circumference.

RULE. The Diameter of a Circle, being 1, the Circumference is 3.1416 *ferè*, and all Circles being to one another as their Diameters; therefore, as 1 : 3.1416 : : ſo is the Diameter of any Circle to its Circumference.

Exa. What is the Periphery or Circumference of a Circle, whoſe Diameter is 15 (Inches, Feet, Yards, *&c.*)

1 : 3.1416 : : 15 : 47.124 Circumference.

PROP. II.

Having the Circumference of a Circle, to find the Diameter.

RULE. This being the Converſe of the laſt Prop. ſay, As 3.1416 is to 1, ſo is the Circumference given, to the Diameter required.

Exa. What is the Diameter of a Circle whoſe Circumference is 47.124?

3.1416 : 1 : : 47.124 : 15 Diameter.

Or, becauſe 1 divided by 3.1416 quotes .3183, therefore the Circumference of any Circle multiplied by .3183 gives the Diameter, for 47.124×.3183=15 *ferè*, for it is =14.999, *&c.*

PROP.

PROP. III.

Having the Diameter of a Circle, to find the Area.

RULE. Multiply $\frac{1}{4}$ of the Circumference by the Diameter, and the Product is the Anſwer.

Or thus: The Diameter of a Circle being 1, the Area is .7854 *ferè*; wherefore, multiplying the Square of the Diameter of any Circle by the foreſaid Number, you have the Area.

Exa. What is the Area of a Circle whoſe Diameter is 15?

By Prop. I. the Circumference is 47.124, wherefore, $\frac{47.124}{4} \times 15 = 11.781 \times 15 = 176.715$ Area.

By the ſecond Method $15 \times 15 = 225$, and $225 \times .7854 = 176.715$, as before.

PROP. IV.

Having the Area of a Circle, to find the Diameter.

RULE. Divide the given Area by .7854, and the Quote is the Square of the Diameter, whoſe Root is the Thing ſought.

Exa. What is the Diameter of a Circle, whoſe Area is 176.715?

.7854)176.715(225 Square of the Diameter, whoſe Root is 15.

PROP. V.

Having the Circumference of a Circle, to find the Area.

RULE. Say, As 1 is to .079577, ſo is the Square of the Circumference to the Area.

But this being too tedious, find the Diameter by Prop. 2d, and thence the Area by Prop. 3d.

Exa. What is the Area of a Circle whoſe Circumference is 47.124.

By the firſt Method 47.124 ſquared, is 222.671376, which

which multiplied by .079577 produces 176.698822, *&c.* for the Area.

By Prop. 2. the Diameter is found to be 15, and by Prop. 3. the Area is 176.715.

PROP. VI.

Having the Area, to find the Circumference.

RULE. Divide the given Area by .079577, and the Quote is the Square of the Circumference, whose Root is the Thing sought.

Exa. What is the Circumference of a Circle whose Area is 176.715 ?

$\frac{176.715}{.079577} = 2220.68$, whose Root is 47.124.

PROP. VII.

To find the Area of a Semicircle, as *abdemc*, *Fig.* 1.

RULE, Multiply the Square of the Diameter by .3927 (*viz.* $\frac{1}{2}$ of .7854) and the Product is the Answer.

PROP. VIII.

To find the Area of a Quadrant, as *acdb*, *Fig.* 1.

RULE. Multiply the Square of twice the Radius by .19635, (*viz.* $\frac{1}{4}$ of .7854) and the Product is the Answer.

PROP. IX.

To find the Area of a Sector and Segment of a Circle, as the Sector *bced*, and the Segment *bed*, *Fig.* 1.

In order to find the Area of the Segment of a Circle, there must always be given the Circle's Diameter, or else its Circumference or Area, to find the Diameter; and 2dly, the Segment's Base, otherwise called the Chord, as *be*, or the vers'd Sine *dh*, which is the Segment's Height: Then 'tis plain, that if the Area of the Triangle *bce*, be taken from the Area of the Sector *bced*,

bced, the Remainder will be the Area of the Segment *bhed*; and if the Area of the Segment *bhed* be taken from that of the whole Circle, the Remainder will be that of the other Segment *bakme*.

Let the Diameter *am* be 15
Then *cd* is - - 7.5
Let *dh* be - - 3
Then *hc* is - 4.5

and *bh* is found by ſubtracting the Square of *hc*, *viz.* 20.25 from the Square of *bc*, *viz.* 56.25; and taking the Square Root of the Remainder, *viz.* 6=*bh*. Now, the Arch *bd*, or the Quantity of the Angle *bcd*, muſt be found by Trigonometry, thus, *bc*: *bh*:: Radius: Sine of *c*; that is, 7.5 : 6 :: 10.0000 : 53° 7′; Then, as the Circumference of the whole Circle in Degrees, is to the Circumference in equal Parts, ſo is any Arch in Degrees (here *bd* 53°7′) to the ſame Arch in equal Parts. The Circumference of a Circle whoſe Diameter is 15 is (by Prop. 1.) 47.124: therefore, 360° : 47.124 :: 53°7′ : 6.948=*bd*, conſequently 6.948 ×7.5=52.11 (found by Prop. 18. Sect. 1.) Area of the Sector *bced*; and *bh* being 6, *be* muſt be 12; wherefore, by the ſame Prop. 18. Sect. 1. 12×2.25=27, Area of the Triangle *bce*; and their Difference is 25.11, for the Area of the Segment *bed*.

The Area of the Segment of a Circle may be otherwiſe found, *viz.*

RULE. Take $2\frac{1}{3}$ times the Square of the Semidiameter of the Circle, from which ſubtract $1\frac{1}{3}$ times the Product of the Semidiameter into the Difference betwixt the Segment's Height and the Circle's Semidiameter, as alſo the Square of the ſaid Difference; the Remainder divide by $1\frac{1}{2}$ times the Semidiameter + the Difference betwixt the Semidiameter and the Segment's Height, which Quote multiply by half the Segment, and the Product is the Segment's Area.

7.5×7.5×$2\frac{1}{3}$=131.25 and 7.5×4.5×$1\frac{1}{3}$=45. Alſo, 4.5×4.5=20.25. The Sum of theſe two laſt is 65.25; therefore

therefore 131.25—65.25=66, which divided by 15.75, *viz.* $1\frac{1}{2}$ times the Semidiameter, *&c.* quotes 4.19, and this multiplied by half the Segment, *viz.* 6, gives 25.14 for the Segment's Area.

PROP. X.

To find the Area of the Lune, or ſuch a Part of a Circle as *armca.*

RULE. Find the Area of the Semicircle *abdemc*, from which ſubtracting that of the Segment *armca*, the Remainder is the Area of the Lune *armca.*

PROP. XI.

To find the Area or ſuperficial Content of a Square.

A Square is a Figure conſiſting of 4 equal Sides, and as many Right Angles, as ABCD, *Fig.* 2.

RULE. Multiply the length of a Side by itſelf, and the Product is the Anſwer.

Exa. How many Square Feet are contained in a Table, each of whoſe Sides is 3 Feet 6 Inches, or 42 Inches?

$3.5 \times 3.5 = 12.25 = 12\frac{1}{4}$ Sq. Feet.

Or, $3\frac{1}{2} \times 3\frac{1}{2} = 12\frac{1}{4}$ as before: For

		Or thus:	
3	6		42
	$3\frac{1}{2}$		42
10	6		84
1	9		168
12	3		144)1764($12\frac{1}{4}$ as before.
			1728
			36

Exa. 2. There is a ſquare Piece of Ground, the length of each of whoſe Sides is 136.25 Yards, what is the Content in Acres?

136.25×136.25=18564.0625, which divided by 4840,
A. R. P. Yds. Feet. Inch.
quotes 3.83555 Acres=3. : 3 : 13 : 20 : 7 : 44

PROP. XII.

To find the Area or ſuperficial Content of a Rectangle, Parallelogram or Oblong, *Fig.* 3.

This Figure conſiſts of 4 Right Angles, having its oppoſite Sides equal.

RULE. Multiply the Length by the Breadth, and the Product is the Area ſought.

Exa. There is a Table in form of a long Square or Oblong, whoſe Length is 5 Feet 4 Inches, and Breadth 3 Feet 8 Inches: How many ſquare Feet doth it contain?

Feet. Inch.
4 : 5=53 *Inch.*
3 : 8=44 *Inch.* Then, 53×44=2332 ſq. Inches, which divided by 144 (the ſq. Inches in a ſq. Foot)= 16 ſq. Feet, and 28 ſq. Inches, *Anſr.*

Or thus; 4 : 5
3 : 8

13 : 3 for 3
2 : 2.5 for 6 Inches.
0 : 8.833 for 2 Inches.

16 : 2.333, *&c.*

that is, 16 ſq. Feet $+\frac{2}{12}$ or $\frac{1}{6}$ of a ſq. Foot $+\frac{333, \&c.}{1000, \&c.}$ Parts of $\frac{1}{12}$ of a ſquare Foot. So that the Anſwer is $16\frac{1}{3}$ ſq. Feet *ferè*.

Exa. 2. There is a Room in form of a long Square, whoſe Length is 20 Feet and Breadth 15: How many Yards of 3 Quarter Broad Cloth will be ſufficient to hang the ſame, its Height being 7 Feet?

20+15=35 and 35x2=70 and 70x7=490= the ſq. Feet contained in the 4 Walls=70560 ſq. Inches, and 27x36=972 the ſq. Inches contained in 1 Yard;

Yds. Qrs. Nails.

wherefore 972)70560(72—2—1 *Anſr.*

PROP. XIII.

To find the Area or ſuperficial Content of a Rhombus or Rhomboides, *Fig.* 4 and 5.

A Rhombus has 4 equal Sides, whereof the two oppoſite Sides are equal, and the Sides parallel, and may be repreſented by a Diamond, or a Square out of its true Poſition, as *Fig.* 4.

A Rhomboides has 4 equal Sides, whereof the two oppoſite are equal and parallel, and the oppoſite Angles equal, being a Parallelogram out of its true Poſition, as *Fig.* 5.

RULE. In either of theſe Figures, multiply the Baſe into the perpendicular height, and the Product is the Anſwer.

Exa. There is a piece of Wainſcot in form of a Rhombus, whoſe Baſe or Length is 2 $\frac{1}{2}$ Feet, and the Height or Perpendicular CH, 2 Feet 4 Inches; What is the Area in ſquare Feet?

2$\frac{1}{2}$ Feet = 30 Inch.

2$\frac{1}{3}$ Feet = 28 Inch. Then 30x28=840, and this divided by 144 quotes 5$\frac{10}{12}$ ſq. Feet = 5 ſq. Feet and 120 ſq. Inches.

Or thus;	Or thus;
2 : 4	2 : 4
2 : 6	2$\frac{1}{2}$
4 : 8	4 : 8
1 : 2	1 : 2
5 : 10	5 : 10

PROP. XIV.

To find the Area or ſuperficial Content of a Trapezium.

This Figure conſiſts of 4 Sides and 4 Angles, which are generally neither parallel nor equal, as *Fig.* 6.

RULE. Divide it into two Triangles, by drawing a Diagonal Line from one of its acute Angles to the oppoſite one, and thereon letting fall 2 Perpendiculars from the oppoſite Angles, multiply half the Length of the Diagonal into the Sum of the Perpendiculars (or the Length of the Diagonal into half the Sum of the Perpendiculars) and the Product is the Area.

Exa. Suppoſe the Diagonal AC 40 Yards, the Perpendicular P*p* 19 Yards, and the other Perpendicular PP 12 Yards; What is the Area of the Figure?

$\frac{40}{2}=20$; and $19+12=31$. Then $20\times31=620$ ſq. Yards for the Anſwer.

PROP. XV.

To find the Area of any regular Polygon, as *Fig.* 7.

A Regular Polygon is a Figure conſiſting of more than 4 equal Sides.

RULE. Circumſcribe it with a Circle touching the angular Points; and from the Centre let fall a Perpendicular (on any ſide) half of which multiplied into the Sum of the Sides gives the Area.

Exa. Required, the Area of the Pentagon ABHDE, each of whoſe Sides is 75 Yards, and the Perpendicular CG 51 Yards? $75\times5=375$, Sum of the Sides, and 25.5 half the Perpendicular; wherefore, $375\times25.5=9562.5$ ſquare Yards, which divided by 4840, the ſquare Yards in an Acre, quotes 1.976 *ferè* Acres for the Anſwer.

PROP.

PROP. XVI.

To find the Area or ſuperficial Content of an irregular Polygon, as *Fig.* 8.

An irregular Polygon is a Figure conſiſting of more than 4 Sides, all of which, as alſo the Angles, are generally unequal.

RULE. Reduce it into Triangles by drawing Diagonals, and the Sum of their Areas is the Area required.

Exa. Required the Area of the irregular Polygon, HIKLM, the Length of the Side HI=5, IK=$5\frac{1}{2}$, KL=$8\frac{1}{2}$, LM=9, and MH=7, and the Perpendicular *Ii*=4.5, M*m*=6, and K*k*=6.5.

3.5×4.5=15.75, Area of the Triang. HIM.
2.75×6=16.5 Area of the Triang. IMK.
4.5×6.5=29.25 Area of the Triang. KML.

61.5 Area of the whole Fig. HIKLM.

PROP. XVII.

To find the Area of a Parallelopleuron, as *Fig.* 9.

This Figure conſiſts of 4 Sides, having two oppoſite ones parallel, and the 2 Angles at each end equal to one another.

RULE. Draw a Diagonal, on which letting fall 2 Perpendiculars, by $\frac{1}{2}$ of their Sum multiply the Diagonal, and the Product is the Area.

Exa. Suppoſe the Diagonal AB 36, the Perpendicular P*p* 21, and the other Perpendicular *pp* 13.21+13 =34 and 34÷2=17; and 36×17=618 for the Area of the Figure A*p*BP.

PROP. XVIII.

To find the Area or ſuperficial Content of a Triangle, as *Fig.* 10.

RULE. Of what kind ſoever it be, multiply the Length of the Baſe by half the Length of the Perpen-

dicular, or Height: Or, multiply the Length of the Perpendicular by half the Length of the Base, and you have the Square Content sought; for every Triangle is half its circumscribing Parallelogram.

Exa. In the Triangle ABC, suppose the Base BC 43, and the Perpendicular AD 27; What is the Area?

$\frac{43}{2}$=21.5 and 21.5×27=580.5 Area sought.

PROP. XIX.

To find the Area or superficial Content of an Ellipse or Oval, as *Fig.* 11.

An Oval or Ellipse is formed by cutting a Cone through slaunt-wise, or not perpendicular to the Base.

RULE. Multiply the greatest Diameter by the least, and the Product by .7854, and this last Product is the Area required.

Exa. Suppose in the Ellipsis ABCD, the greatest Diameter AC is 41, and the least Diameter BD 28; What is the Area?

41×28=1148 and 1148×.7854=901.6392 for the Area or square Content.

PROP. XX.

To find the Area of a Parabola, as *Fig.* 12.

This Figure is form'd by cutting a Cone parallel to its opposite Side.

RULE. Multiply the Height by the greatest Breadth, and $\frac{2}{3}$ of the Product is the Answer.

Exa. Suppose the Height AB 46, and the greatest Breadth CD 32, what is the Area?

46×32=1472, which multiplied by 2, and the Product divided by 3, quotes 981 $\frac{1}{3}$ for the Answer.

PROP. XXI.

To find the Area or superficial Content of a Cube.

A Cube is a Solid, bounded by 6 equal Squares in form of a Die.

RULE.

RULE. Find the Area of any of the Sides by Prop. 11, and multiply it by 6 for the Anſwer.

Exa. Suppoſe a Cube having its Side 16, what is the ſuperficial Content?

16×16=256 and 256×6=1536 *Anſr.*

PROP. XXII.

To find the Area or ſuperficial Content of a Globe or Sphere.

A Globe or Sphere is a Solid, bounded or included within one regular Superficies; and is formed by the Rotation of a Semicircle about its Diameter.

RULE. Multiply the Diameter or Axis into the Circumference of a great Circle upon it: Or, multiply the Area of a great Circle upon it by 4; and the Product is the Anſwer.

Exa. What is the Area of a Sphere or Globe whoſe Diameter or Axis is 16 Inches, Feet, *&c*?

By Prop. 1. the Circumference of a great Circle upon it is 50.2656, which multiplied by the Diameter 16, gives 804.2496, for the ſuperficial Content required.

By the other Method, I find the Area of a great Circle upon it (Prop. 3.) to be 201.0624, which multiplied by 4, gives 804.2496, as before.

PROP. XXIII.

To find the Area of a Fruſtum of a Globe or Sphere, as the Fruſtum ACF, *Fig.* 13.

RULE. Firſt, find the Altitude of the other Fruſtum, by dividing the Square of the Semidiameter of the given Fruſtum's Baſe by its Altitude; which add to the given Fruſtum's Altitude; then ſay, As the Axis of the whole Sphere is to its Superficies, ſo is the Height or Axis of the Segment or Fruſtum given, to the Curve Superficies thereof; to which adding the Area of the Segment's Baſe, the Sum is the ſuperficial Content of the whole Segment.

Exa. What is the Superficial Content of a Fruſtum of a Sphere, whoſe Altitude is 6 Inches, and the Diameter of the Fruſtum's Baſe 24 Inches?

The Square of the Semidiameter of the Fruſtum's Baſe (12×12) is 144, which divided by 6 (the Fruſtum's Altitude) makes 24 Inches for the Altitude of the other Fruſtum; which added to 6, makes 30 for the Axis or Diameter of the whole Sphere, whoſe Superficial Content is found to be (by laſt Prop.) 2827.44; therefore, 30: 2827.44 :: 6 : 565.488, the Curve Superficies of the Fruſtum, to which adding the Area of its Baſe 452.3904 (found by Prop. 3.) gives 1017.8784 for the Anſwer or Area of the Fruſtum.

PROP. XXIV.

To find the Area or Superficial Content of a Priſm.

A Priſm is a Solid, contained under ſeveral Planes, two of which being oppoſite, *viz.* the two Ends, are called the Baſes, and theſe are parallel and equal, and the other Planes are Parallelograms, in which a Right Line may be every where applied from Baſe to Baſe.

Priſms are either triangular, multangular, circular or elliptical, *&c.* according to the Figure of the Baſe; thus a Cube is a Priſm, bounded by 6 equal Square Planes; a Parallelopipedon is a Priſm having its Sides bounded by 4 equal Parallelograms, and 2 Square Baſes or Ends; a Cylinder (or Solid, like a Rolling Stone in a Garden) is a circular or round Priſm.

Now, to find the Area or ſuperficial Content of any triangular or multangular Priſm:

RULE. Take the Sum of the Areas of the quadrilateral Figures, which terminate or bound it, and you have the Area required.

Exa. There is a triangular Priſm, *i. e.* a Priſm having a Triangle for its Baſe, the Length of the (Baſe's) Side 12 Inches, and the Priſm's Height 30 Inches; what is the Area or ſuperficial Content?

12×30=360 Area of one Side; and 360×3=1080 Area

rea of all the ſides. Then to find the Area of the Baſe, ſuppoſe the Perpendicular let fall from any one Angle upon the oppoſite Baſe to be 10.3, wherefore 10.3×6 =61.8, which doubled, makes 123.6 for the Areas of both Baſes, and 1080+123.6=1203.6 ſq. Inches for the Area or ſuperficial Content of the whole Priſm.

Exa. 2. There is a multangular Priſm, having for its Baſe a Polygon of 8 equal Sides, each of which is 2 Feet, and the Height 2½ Feet; What is the Area or ſuperficial Content?

By Prop. 12; 24×30=720 and 720×8=5760 Area of all the Sides. Then to find the Area of the Baſe, ſuppoſe a Perpendicular let fall from the Center of its inſcribed or circumſcribed Circle on any of the Sides to be 29; wherefore, by Prop. 15; 14.5×8=116, which doubled, makes 232 for the Area of both Baſes; and 5760+232=5992 ſq. Inches for the Area or ſuperficial Content of the whole Priſm.

PROP. XXV.

To find the Area or ſuperficial Content of a circular Priſm or Cylinder.

RULE. Multiply the Circumference of one of the Baſes into the length of the Cylinder, and to the Product add the Area of both Baſes.

Exa. There is a Cylinder, whoſe Length is 3 Feet, and Diameter of the Baſe 10 Inches, what is the ſuperficial Content?

By Prop. 1. the Circumference of the Baſe is 31.416, which multiplied into the Length, 3 Feet =36 Inches, makes 1130.976 ſq. Inches for the Curve Superficies of the Cylinder.

Then the Area of the Baſe is found by Prop. 3d to be 78.54, which doubled, gives 157.08 for the Area of both Baſes; wherefore 1130.976+157.08=1288.056 ſq. Inches for the Anſwer.

PROP.

PROP. XXVI.

To find the Area or ſuperficial Content of a Pyramid.

Solids, which decreaſe gradually from the Baſe till they come to a Point, are in general called Pyramids, and are of different kinds according to the Figure of the Baſe. Thus, a Pyramid having a Triangle for its Baſe, is called a Triangular Pyramid. If the Baſe is a Parallelogram, it is called a Parallelogramic Pyramid; and if a Circle, it is called a Circular Pyramid, or ſimply a Cone. The Point in which the Pyramid ends is called the Vertex, and a Line drawn from the Vertex perpendicular to the Baſe, is called its Height.

And firſt, to find the Area of a circular Pyramid or Cone.

RULE. Multiply the Circumference of the Baſe into ½ the Length of the Side, (not the Perpendicular Height) and the Product is the Area required, when thereto you have added the Area of the Baſe.

Exa. There is a Cone whoſe Diameter at the Baſe is 16 Inches, and Length of the Side 48 Inches; What is the ſuperficial Content? *Fig.* 14.

3.1416×16=50.2656 Circumference of the Baſe, which multiplied by 24, *viz.* half the Length of the Side, gives 1206.3744 for the Area of the Curve Superficies. Then to find the Area of the Baſe 16×16=256, and 256×.7854=201.0624. Laſtly, 1206.3744+201.0624=1407.4368 ſq. Inches.

Or thus, As the Semidiameter of the Baſe is to the Length of the Side, ſo is the Area of the Baſe to the Area of the Convex Superficies, to which adding the Area of the Baſe, as by the firſt Rule, the Sum is the ſuperficial Content of the whole.

PROP.

PROP. XXVII.

To find the Area of all other Sorts of Pyramids.

RULE. Take the Sum of the Triangular Figures which conſtitute the Pyramid, and thereto adding the Area of the Baſe, the Sum is the Area ſought.

Exa. There is a Pyramid, having for its Baſe a Triangle, one Side of whoſe Baſe AC is 18 Inches, CB 16, and AB 20, and the Length of the Pyramid's Side 48 Inches; what is the Area or ſuperficial Content?

By Prop. 18th, 18×24=432; 16×24=384, and 20×24=480, whoſe Sum is 1296 (or 18+16+20=54, and 54×24=1296) for the Content of the three Sides of the Pyramid. Then to find the Area of the Baſe, ſuppoſe the Perpendicular CD let fall on the Baſe AB to be 13.75, by the ſame Prop. $20\times\frac{13.75}{2}$ or 10×13.75=137.5; to which adding 1296, the Sum is 1433.5 ſq. Inches for the Anſwer.

PROP. XXVIII.

To find the Area of the Fruſtum of a Cone cut by a Plane parallel to its Baſe. *Fig.* 14.

RULE. Add to the ſuperficial Content of the whole Cone twice the Area of the Baſe of the ſmall Cone, and from that Sum taking the ſuperficial Content of the ſmall Cone, you have the Anſwer or ſuperficial Content of the Fruſtum.

Exa. Let ABCD repreſent the Fruſtum of a Cone, the Diameter of whoſe greater Baſe AB is 16 Inches, and that of the leſſer 6 Inches, and Length of the Side of the Fruſtum 30 Inches.

Suppoſe the Height of the Side of the whole Cone 48 Inches (which is eaſily found, either by laying down the Pyramid by Scale and Compaſſes, or by applying Lines to the Solid or oppoſite Sides, and meaſuring from the Vertex where they meet, to the Circumference,

rence of the Baſe) then by Prop. 27, the ſuperficial Content of the whole Cone is 1407.4368 ſq. Inches. Then to find the Area of the Baſe of the ſmall Cone (or Top of the Fruſtum) 6×6=36, and 36×.7854= 28.2744, which doubled, makes 56.5488, and this added to 1407.4368, makes 1463.9856. Next, the Area of the ſmall Cone is found (by Prop. 27.) to be 197.9208; therefore 1463.9856 — 197.9208=1266.0648 for the Area of the Fruſtum ACDB.

After the ſame Method is found the ſuperficial Content of the Fruſtum of any other kind of Pyramids.

SECT. II. *Of Solids.*

PROP. I.

To find the Solidity of a Priſm.

RULE. Multiply the Area of the Baſe into the Priſm's Height, and the Product is the ſolid Content.

Exa. There is a Triangular Priſm, having the Side of the Baſe 12 Inches, and Height 30 Inches; What is the Solidity in Inches?

By Prop. 18. Sect. 1. the Area of the Baſe is 61.8, which multiplied by the Height 30, gives 1854 ſolid or cubic Inches for the Anſwer; and this divided by 1728 (the ſolid Inches in 1 Foot) produces 1 ſolid Foot and 126 ſolid Inches.

Exa. 2. There is a circular Priſm, whoſe Length is 5 Feet, and Diameter of the Baſe 10 Inches; What is the Solidity?

By Prop. 3. Sect. 1. the Area of the Baſe is 78.54, which multiply by 60 Inches = 5 Feet, makes 471.24 cubic or ſolid Inches for the Anſwer.

Exa.

Exa. 3. There is an Elliptical Priſm, the greateſt Diameter of whoſe Baſe is 41 Inches, and the leaſt 28, the Length of the Priſm being 6 Feet; What is the Solidity in Inches?

By Prop. 19. Sect. 1. the Area of the Baſe is 901.6392, which multiplied into the Height 72, gives 64918.0224 ſolid Inches = 37 ſolid Feet, 982.0224 ſolid Inches.

PROP. II.

To find the Solidity of a Pyramid.

RULE, Multiply the Area of the Baſe by $\frac{1}{3}$ of the Perpendicular, Height, and the Product is the Anſwer: for every Pyramid is $\frac{1}{3}$ of a Priſm, having the ſame Baſe and Height.

N. B. To find the perpendicular Height, having the Height of the Side, from the Square of the Side-Height, ſubtract the Square of the Semidiameter of the Baſe, and the Square Root of the Remainder is the perpendicular Height.

Exa. There is a Pyramid, having a Circle for its Baſe, whoſe Diameter is 16 Inches, and Height of the Side 48 Inches.

Firſt, the perpendicular Height is 47.328, then by Prop. 3. Sect. 1. the Area of the Baſe is 201.0624, which multiplied by $\frac{47.328}{3}$ = 15.776, produces 3171.96 cubic Inches, or one ſolid Foot and 1444 ſolid Inches *ferè*.

Exa. 2. There is a Pyramid having a ſquare Baſe, each of whoſe Sides is 12 Inches, and the perpendicular Altitude 40 Inches; What is the Solidity?

12×12=144 the Area of the Baſe; and 144×40=5760 Cubic Inches, which divided by 3 quotes 1920 for the Anſwer.

PROP.

PROP. III.

To find the Solidity of the Fruſtum of a Cone, cut by a plain Parallel to its Baſe, *Fig.* 14.

RULE. Find the Solidity of the whole Cone by laſt Prop. and from thence taking the Solidity of the ſmall Cone, the Remainder is the Solidity of the Fruſtum.

Exa. Let ABCD repreſent the Fruſtum of a Cone, the Diameter of whoſe greater Baſe AB is 16 Inches, and that of the leſſer CD 8 Inches, and Length of the Side 30 Inches; What is the Fruſtum's Solidity?

Firſt, 30×30=900 ſquare of the Side-Height, and 64 (*viz.* 8×8) the ſquare of the Semidiameter of the Baſe; then 900—64=836, whoſe ſq. Root is 28.914 *ferè* the Height of the Fruſtum.

Now, to find the Height of the whole Cone, and conſequently its Solidity, ſay, 4.8 : 8 :: 28.914 : 48.18, that is, as the Difference betwixt the Fruſtum's greateſt and leaſt Semidiameters is to the greateſt Semidiameter, ſo is the Fruſtum's Height to that of the whole Cone.

By Prop. 2d, the Solidity of the whole Cone is 3229.06214, and by the ſame Prop. the Solidity of the ſmall Cone is 206.59574, wherefore 3229.06214 —206.59574=3022.4664 for the Solidity of the Fruſtum.

The Solidity of the Fruſtum of a Cone may be otherwiſe found, *viz.* Find the Area of both Baſes, and take a Geometrical Mean betwixt theſe two Areas; which Mean add to the Sum of the Areas, and multiplying the Total by $\frac{1}{3}$ of the Fruſtum's Height, the Product is the Anſwer. Now, to find a Geometrical Mean between any two Numbers, you muſt take the ſquare Root of their Product.

Or thus, Square the greateſt Diameter, as alſo the leſſer, and multiply the two Diameters together, the Sum of which three Numbers multiply by the Height, and the Product divide by 3.8197; or that Sum multiplied

tiplied into $\frac{1}{3}$ of the Height, and the Product by .7854, gives the Solidity.

If ſuch Fruſtums are cut thro' the Extremities of the Baſes by a Diagonal Line, they are called Hoofs, and the Solidity is found, if it be a Cone, by ſquaring the greater Diameter, and adding thereto $\frac{1}{2}$ the Rectangle of the two Diameters, more the Difference betwixt the Diameters, and multiplying the Sum by the Height, and laſtly dividing the Product by 3.8197. Or, by multiplying the ſaid Sum into $\frac{1}{3}$ of the Height, and multiplying the Product by .7854 for the greater Hoof *Fig.* 15.

And for the Content of the leſſer Hoof, ſquare the leſſer Diameter, and take $\frac{1}{2}$ the Rectangle of the two Diameters, from the Sum of which ſubtract the Difference betwixt the 2 Diameters, and the Remainder multiply and divide as before.

And if it is a ſquare Pyramid,

Take the Square of the greater Diameter, $\frac{1}{2}$ the Rectangle of the two Diameters, as alſo the Difference betwixt the Diameters, multiply the Sum of theſe three Numbers by $\frac{1}{3}$ of the Height for the greater Hoof

And take the Square of the leſſer Diameter, and $\frac{1}{2}$ the Rectangle of the two Diameters, from the Sum of which ſubtract the Difference betwixt the two Diameters, and multiply the Remainder by $\frac{1}{3}$ of the Height for the Content of the leſſer Hoof.

PROP. IV.

To find the Solidity of a Globe or Sphere.

RULE. Multiply the Superficies by $\frac{1}{6}$ of its Axis: Or, 2. Multiply the Cube of the Axis by .5236: Or, 3. Becauſe every Globe is $\frac{2}{3}$ of a Cylinder of the ſame Height and Diameter of the Baſe, with the Globe's Axis, multiply the Area of a great Circle by the Diameter, and take $\frac{2}{3}$ of the Product for the Anſwer.

Exa. What is the Solidity of a Sphere, whoſe Diameter is 16?

By Prop. 23. the ſuperficial Content is 804.2496, which multiplied by 2.6666, &c. (*viz.* $\frac{4}{6}$ of the Axis) gives 2144.66, &c. for the Solidity.

2. The Cube of the Diameter is 4096, which multiplied by .5236 gives 2144.66, &c. as before.

3. The Area of a great Circle upon it is 201.0624, which multiplied by the Diameter 16, makes 3216. .9984, $\frac{2}{3}$ of which is 2144.66, &c. as before.

PROP. V.

To find the Solidity of the Fruſtum of a Globe, having the Diameter of the Fruſtum's Baſe and its Altitude.

RULE. Multiply the Square of half the Diameter of the Baſe by 3 times the Altitude, and to the Product add the Cube of the Altitude, the Sum multiply by .5236, and the Product is the Anſwer. *Fig.* 13.

Exa. There is a Fruſtum of a Globe, the Diameter of whoſe Baſe is 24, and its Altitude 6, what is the Solidity?

12×12=144 Square of half the Diameter, and 18 = 3 times the Altitude. Then 144×18=2592; and the Cube of the Altitude is 216 (=6×6×6) wherefore 2592+216=2808. Laſtly, 2808×.5236=1470.2688 for the Anſwer or Solidity of the Fruſtum.

PROP. VI.

To find the Solidity of a Fruſtum of a Sphere, having the Sphere's Axis and Height of the Fruſtum.

RULE. From the triple Product of the Axis into the Square of the Fruſtum's Height, ſubtract twice the Cube of the Height, and multiply the Remainder by .5236 for the Anſwer.

Exa. There is a Fruſtum of a Sphere, whoſe Axis is 30 Inches, and the Fruſtum's Height 6 Inches, what is the Solidity?

36×30×3=3240 triple Product of the Axis into the Square of the Fruſtum's Height, and 432 = twice the Cube

Cube of the Height; wherefore 3240—432=2808 and 2808X.5236=1470.2688 as before.

And the Content of the one Fruſtum taken from that of the whole Sphere, leaves the Content of the other Fruſtum.

If having the Altitude of the Fruſtum with the Diameter of its Baſe, you want to know the Height of the other Fruſtum, or the Sphere's Axis, divide the Square of the Semidiameter of the one Fruſtum's Baſe by the Height of the Fruſtum, and the Quote is the Height of the other Fruſtum, whoſe Sum is the Sphere's Axis.

PROP. VII.

To find the Solidity of the middle Zone of a Sphere. *Fig.* 13. as CDGHEF.

RULE. To twice the Square of the Diameter or Sphere's Axis add the Square of the Diameter of the Baſe, and dividing the Sum by 3.8197, multiply the Quote by the Zone's Height for the Anſwer.

Exa. Suppoſe the Diameter DE or AB 30, the Diameter of the Baſe 24, and the Zone's Height 18; What is the Solidity?

30X30X2=1800 twice the Square of the Diameter; and 24X24=576 ſquare of the Diameter of the Baſe. Then 1800+576=2376, and 2376 divided by 3.8197, quotes 622,038, which multiplied by the Zone's Height 18, the Product is 11196.684 for the Anſwer.

Or from the Solidity of the whole Sphere ſubtract that of twice the Fruſtum CAF, and the Remainder is the Solidity of the middle Zone.

By Prop. 4. the Solidity of a Sphere whoſe Axis is 30 Inches, will be found to be 14137.2, from which ſubtracting twice the Fruſtum CAF, found by Prop. 5, *viz.* 1470.2688X2=2940.5376, the Remainder is 11196.6624, the ſame as before, *ferè*.

PROP. VIII.

To find the Solidity of a Spheroid, *Fig.* 16.

A Spheroid is a Solid, formed by the Rotation of the Semi-ellipſis ABC, about its tranſverſe Diameter AC, which is called the Spheroid's Axis: This Body much reſembles the Shape of an Egg.

RULE. Every Spheroid being $\frac{2}{3}$ of its circumſcribing Cylinder, whoſe Baſe is = to the greateſt Diameter, and its Height that of the Spheroid; therefore find its Content by multiplying the Area of its greateſt Circle by $\frac{2}{3}$ of its Axis.

Exa. What is the Solidity of a Spheroid, whoſe Axis is 40 Inches, and greateſt Diameter 22 Inches?

By Prop. 3. Sect. 1. the Area of its greateſt Circle is 380.1336, which multiplied by 40 gives 15205.344, two third Parts whereof is 10136.896 for the Anſwer.

PROP. IX.

To find the Solidity of the middle Zone of a Spheroid, as *abdc*. *Fig.* 16.

RULE. To twice the Square of the Diameter DB, add the Square of the Diameter of the Baſe *ab* or *cd*, and divide the Sum by 3.8197, the Quote of which Diviſion multiplied by the Height produces the Anſwer.

Exa. Suppoſe the Height of the middle Zone 24 Inches, the Diameter of the Baſe 17 Inches, and the greateſt Diameter 22 Inches; What is the Solidity?

22×22=484; and 484×2=968

17×17=289. Their Sum is 1257, which divided by 3.8197, quotes 329.057, and this multiplied by the Height 24, gives 7897.368 for the Anſwer.

PROP. X.

To find the Solidity of any other Fruſtum of a Spheroid, as A*ab*.

RULE. Find its Solidity as if it was a Sphere, and ſay,

ſay, As the Solidity of the whole Sphere is to the Solidity of the whole Spheroid, ſo is any Part of the Sphere to the like Part of the Spheroid.

Exa. Suppoſe the Height of the Fruſtum 6, and the Diameter of the Baſe 16; What is the Solidity?

By Prop. 5th of this Section, the like Fruſtum of a Sphere would be 716.2848, and by Prop. 4. the Solidity of the whole Sphere is found to be 2423.878; therefore, *&c.*

PROP. XI.

To find the ſolid Content of a Parabolic Conoid, *Fig.* 12.

This Solid is form'd by the Rotation of a Semiparabola CAB, about its Axis AB.

RULE. Multiply the Area of the Baſe by ½ the Altitude, and the Product is the ſolid Content; for every parabolic Conoid is equal to ½ its circumſcribing Cylinder.

Exa. There is a parabolic Conoid, whoſe Baſe is 32 Inches, and its Height 46 Inches; What is the Solidity? .7854×1024=804.2496 Area of the Baſe, which multiplied by 23 (*viz.* ½ the Altitude) gives 18497.8408.

PROP. XII.

To find the Solidity of the lower Fruſtum of a parabolic Conoid, as EFCD. *Fig.* 12.

RULE. Square the Diameter at each end, and multiply their Sum by ½ the Height, and divide the Product by 3.8196.

Exa. Suppoſe EF 24, CD 32 and *b*B 21.3; What is the Solidity?

32×32=1024, and 24×24=576; their Sum is 1600, which multiplied by 31.95 (*viz.* ⅔ of 21.3) gives 51120, and this divided by 3.8196 quotes 13383.6 for the Anſwer.

PROP. XIII.

To find the Solidity of a Parabolic Spindle or Pyramidoid.

This Solid is formed by turning the Parabola about its Baſe. *Fig.* 17.

RULE. Find the Contents as if it was a Cylinder, and take $\frac{8}{15}$ of that Content for the Content of the Pyramidoid.

Exa. Suppoſe the Length AB 66 Inches, and the greateſt Diameter CD 27; What is the Solidity?

By Prop. 1. Sect. 2. The Contents of a Cylinder of the above Dimenſions, is found to be 37788.7356, which multiplied by 8, and the Product divided by 15, gives the Content of the Pyramidoid, *viz.* 20153.9256.

PROP. XIV.

To find the Solidity of the middle Fruſtum of a Parabolic Spindle, ſuch as *aCcdDba*. *Fig.* 17.

RULE. Square the Diameter CD, which double, and thereto add the Square of the Diameter at the Baſe *ab* or *cd*, from the Sum of which take $\frac{4}{10}$ of the Square of the Difference betwixt the 2 Diameters, and dividing the Remainder by 3.8197, the Quote multiplied into the Height is the Solidity required.

Exa. Suppoſe the Diameter CD 27, as before, the Length *mm* 33, at the Diameter at either Baſe *ab* or *cd* 20; What is the Solidity?

27×27=729 and 729×2=1458. Then 20×20=400, their Sum is 1858. Alſo 27—20=7, and 7×7×$\frac{4}{10}$=19.6. Then 1858—19.6=1838.4, which divided by 3.8197 quotes 481.294, and this multiplied by the Length 33, gives 15882.702 for the Anſwer.

PROP. XV.

To find the Solidity of a Cylindroid.

This Figure has an elliptical or oval Baſe, whereas a Cylinder has a circular one.

RULE.

RULE. Find the Area of the Baſe by Prop. 19. Sect. 1. and this multiplied into the Height gives the Anſwer.

Exa. Suppoſe the greateſt Diameter of the Baſe 41, and the leſſer Diameter at the Baſe 28, and the Height of the Cylinder 72; What is the Solidity?

By Prop. 19th, Sect. 1. the Area of the Baſe is 901.6392, which multiplied by 72 gives 64918.022 for the Anſwer.

PROP. XVI.

To find the Solidity of an hyperbolic Conoid.

This Figure is formed by the Rotation of a Semi-hyperbola ACD about its Axis AC. *Fig.* 18.

RULE. To 6 times the tranſverſe Diameter Ag, add 6 times the intercepted Diameter AC for a Diviſor.

2. To 3 times the tranſverſe Diameter add twice the intercepted Diameter, and by the Sum multiply the Content of a Cylinder, whoſe Height is AC, and the Diameter of the Baſe BD; the Product is a Dividend; which divided by the Diviſor above found, gives the Content of the hyperbolic Conoid ABCD.

This Body is more eaſily meaſured by reducing it to a Cylinder by help of a mean Diameter.

PROP. XVII.

To find the Solidity of any Body by immerſing it into Water.

RULE. Immerſe it in Water in a Parallelopiped, whoſe Sides are exactly divided into Inches and Parts, and the Solidity of the Water raiſed, will be equal to that of the Body immerſed.

Or thus: Fill any Veſſel with Water, and having immerſed the Body, carefully reſerve the Water which flows over into another Veſſel, and find the Solidity of this Water by meaſuring it in a Veſſel of a known Capacity, for the Content or Solidity of this ſhall be that of the Body immerſed.

SECT.

SECT. III. *Of Gauging.*

PROP. I.

TO find the Content in Ale, Wine or Corn Gallons *Engliſh* Meaſure, or in *Scots* Pints, of a Square Ton or Veſſel.

RULE. Multiply the Length or Breadth in Inches by itſelf, and the Product is the Area or Content at one Inch deep, which multiplied by the Height or Depth gives the ſolid Content in Inches, which to reduce to

Ale Gall.	divide by	282.
Wine Gall.		231.
Corn Gall.		268.8
Scots Pints,		102.3

Or multiply the foreſaid Solidity in Inches by .003546; .004329; .003722; .009775 reſpectively, which Multipliers are thus found.

282)1.00000, *&c.* (.003546
231)1.00000, *&c.* (.004329
268.8)1.00000, *&c.* (.003722
102.3)1.00000, *&c.* (.009775

Exa. Suppoſe the Side of a Square Veſſel 40.2 Inches, and Height 10.3; How many Gallons of Ale, Wine or Corn or *Scots* Pints doth it contain?

40.2×40.2=1616.04, which multiplied by the Height 10.3 gives 16645.212, and this divided by 282, or multiplied by .003546 makes 59.024 *ferè* Ale Gallons, which you may alſo reduce to Wine or Corn Gallons, *&c.* by dividing or multiplying, as is before taught.

PROP. II.

To find the Content in Gallons of a Veſſel in form of a Right-angled Parallelogram.

RULE. Multiply the Length by the Breadth, and that Product by the Depth for the ſolid Content in Inches, which reduce to Gallons, as before.

Exa.

Exa. Suppoſe the Length 60 Inches, Breadth 40, and Depth 18; What is the Content in Gallons?

60×40=2400 and 2400×18=43200, which divided by 282, or multiplied by .003546 gives 153.1872 for Ale Gallons.

PROP. III.

To find the Content in Ale, *&c.* Gallons of a Veſſel of a Triangular Form.

RULE. Find the Area of the Baſe by Prop. 18th Sect. 1. which multiplied by the Height gives the ſolid Content in Inches, and this laſt divided or multiplied as before, gives the Content in Gallons.

Exa. Suppoſe the Length of the Baſe of any Triangular Veſſel be 25 Inches, the perpendicular Breadth 15, and the Depth 12; What is the Content in Gallons?

25×7.5=187.5 and 187.5×12=2250 ſolid Inches, which divided by 282 or multiplied by .003546 gives 7.9785 for Ale Gallons: and ſo on for Wine, Corn Gallons and *Scots* Pints.

PROP. IV.

To find the Content in Gallons of a Veſſel in form of any other regular or irregular Figure, the Veſſel being equally wide throughout.

RULE. Divide it into Triangles, and multiply the Sum of their Areas into the Depth, the Product is the ſolid Content in Inches, which reduce to Gallons, by multiplying or dividing as before.

PROP. V.

To find the Content in Gallons of a Veſſel that's circular, and equally wide throughout, *i. e.* in form of a Cylinder.

RULE. Multiply the Area of the Baſe in Inches by the Veſſel's Depth, and the Product is the Solidity

in

in Inches, which reduce to Gallons, *&c.* by dividing or multiplying as before.

Exa. Let the Diameter of a Cylindrical Veſſel be 48.3 Inches and the Depth 60.5; how many Gallons doth it contain?

By Prop. 3. Sect. 1. the Area of the Baſe, is 1832 .2518, which multiplied by 60.5 produces 110851 .234263 ſolid Inches; and this divided by 282, or multiplied by .003546 makes 393.0784, for Ale Gallons, *&c.*

Or thus: Divide the Square of the Diameter by

359.05 for Ale Gall.	The Quotes are the Areas
294.12 for Wine Gall.	at 1 Inch deep, which there-
342.24 for Corn Gall.	fore muſt be multiplied by
130.25 for *Scots* Pints.	the whole Depth for the Anſwer.

N. B. Theſe Diviſors are found by dividing 282, 231, 268.8, 102.3 ſeverally by .7854 the Area of that Circle whoſe Diameter is 1.

PROP. VI.

To find the Content in Gallons of a Veſſel of an Elliptical Bottom or in Form of a Cylindroid equally wide throughout.

RULE. Find its Solidity by Prop. 15. Sect. 2. and divide or multiply as before, in order to reduce it to Gallons, *&c.* Or (which is the ſame Thing) multiply the Length of the Bottom or Top by its Breadth, and that Product by .7854 for the Area at one Inch deep, which multiplied by the whole Depth, and the Product divided by 282, *&c.* gives the Anſwer in Gallons, *&c.*

Exa. Suppoſe the greateſt Diameter of the Top or Bottom 41 Inches, and the leſſer 28, and Height or Depth of the Veſſel 72 Inches; What is the Content in Gallons, *&c.*

By Prop. 15. Sect. 2. the Solidity in Inches is 64918.022, which divided by 282, quotes 226.66 for Ale Gallons.

But

But if the Veſſel, whether circular or elliptical, be wider at the Top than at the Bottom (as they generally are) the beſt Way of finding its Content, is by taking the Sum of the Areas of Top and Bottom, and multiplying ½ the ſaid Sum by the Height or Depth, and dividing as before, in order to reduce it to Gallons.

Exa. What is the Content of a Tub, whoſe Diameter at the Top is 37 Inches, and at the Bottom 30 Inches, the Depth being 24 Inches?

37×37=1369 and 1369×.7854=1075.2126 Area at the Top.

30×30×.7854=706.86 Area at the ott om.

Their Sum is 1782.0726, ½ of which is 891.0363, which multiplied by 24, makes 21384.8712; and this divided by 282, gives 75.8329 Ale Gallons.

PROP. VII.

To find the Content in Gallons of a Priſm, having for its Sides Parallelograms ſtanding at Right Angles with the Baſe.

RULE. Find the Solidity in Inches by Prop. 1. Sect 2. and divide the ſame by 282, *&c.* for the Anſwer in Gallons, *&c.*

Exa. There is a Triangular Priſm, the Length of one of the Sides of whoſe Baſe is 12 Inches, of another 16, and of the other 18, and the Depth 24 Inches; What is the Content in Gallons?

By Prop. 1. Sect. 2. the Solidity of the Priſm, (ſuppoſing the Perpendicular let fall on the greateſt Side of the Baſe, 10.5 Inches) is found to be 2268 ſolid Inches; which divided by 282, quotes 8.04 Ale Gallons.

PROP. VIII.

To find the Content in Gallons of a Veſſel in form of a Pyramid, having Right Lines for its Baſe.

RULE. Find its Solidity in Inches by Prop. 2. Sect. 2. and divide by 282, *&c.* for the Anſwer in Gallons, *&c.*

Exa.

Exa. How many *Scots* Pints doth a Pyramidical Veſſel contain, whoſe Baſe is a Circle of 16 Inches Diameter, and the Perpendicular Height or Depth of the Veſſel 30 Inches?

16×16×.7854×10=2010.624 Solidity in Inches, and this divided by 102.3 or multiplied by .009775, gives 19.65=19 Pints, 2 Mutchkins, 2¾ Gills *ferè*.

If it is a ſquare Pyramid, its Content in Gallons may be thus found; Square the Side of the Baſe and multiply the ſaid Square by the Perpendicular Height, dividing the Product by 3 Times the conſtant Diviſors, 282, 231, 268.8, 102.3 for the Anſwer in Ale, *&c.* Gallons.

PROP. IX.

To find the Content in Gallons of the Fruſtum of a Pyramid cut by a Plain, parallel to its Baſe.

RULE. By Prop. 3. Sect. 2. find its Solidity in Inches, which divide by 282, *&c.* for the Content in Gallons, *&c.*

If the Veſſel is in Form of a Fruſtum of an Elliptical Pyramid, or indeed of any other Form, find the Areas of the Top and Bottom, then a Geometrical Mean betwixt them, and multiply the Sum of theſe two Areas and the Mean by ⅓ of the Fruſtum's Height, and divide the Product by 282, *&c.* for Gallons, *&c.*

PROP. X.

To find the Content in Gallons, *&c.* of a Veſſel in Form of a Globe.

RULE. By Prop. 4. Sect. 2. find the Solidity in Inches, and divide or multiply as before, to reduce it to Gallons, *&c.*

Or thus. Multiply the Cube of the Axis by .001856, or divide by 538.57 for Ale Gallons: multiply by .002266, or divide by 441.17 for Wine Gallons, and the Quotes or Products are the reſpective Contents. For if the Cube of the Axis being multiplied by .5236 gives the Con-

Content in Inches, (Prop. 4. Sect. 2.) therefore by dividing the ſaid Content by 282.231, *&c.* the Quote gives the Content in Gall. but

282) .5236 (.001856 Multiplier for *A.* Gall.
231) .5236 (.002266 Multiplier for *W.* Gall.

Or,

.5236) 282. (538.57 Diviſor for *A.* Gall.
.5236) 282 (441.17 Diviſor for *W.* Gall.

PROP. XI.

To find the Content in Gall. of the Segment of a Spherical Veſſel, having the Diameter of its Baſe and Height.

Rule. Find the Content in Inches by Prop. 5. Sect. 2. and divide or multiply as before, for the Content in Gall.

Or thus, To the triple Square of half the Diameter, add the Square of the Height, which Sum multiply into the Height, and divide the Product by 538.57 or multiply by .001856 for Ale Gall. and divide or multiply by 441.17 and .002266 for Wine Gallons.

PROP. XII.

To find the Content in Ale, Wine, *&c.* Gall. of a Veſſel, in Form of the middle Fruſtum of a Globe.

RULE. By Prop. 7. Sect. 2. find its Solidity in Inches, and multiply or divide as before, for the Anſwer in Gallons.

PROP. XIII.

To find the Contents in Gallons, *&c.* of Veſſels in Form of a Spheroid, middle Zone of a Spheroid, Parabolic Conoid, lower Fruſtum of a Parabolic Conoid, Parabolic Spindle, middle Fruſtum of a Parabolic Spindle, Cylindric and Hyperbolic Conoid.

RULE. Find their Contents in Inches by Prop. 8. 9. 11. 12. 13. 14. 15. 16. Sect. 2. and divide or multiply as before, for the Content in Gallons, *&c.*

PROP. XIV.

To find the Content in Gall. *&c.* of any close Cask, as *Fig.* 19.

The Contents of such Casks cannot be found by any one general Rule; and therefore they are supposed to be either,

1. The middle Zone of a Spheroid, or
2. The middle Zone of a Parabolic Spindle, or
3. The lower Frustums of 2 equal Parabolic Conoids, or
4. The lower Frustums of 2 equal Cones.

1. If the Staves of the Cask have a great Curve as the outward Lines of the Figure, it is supposed to be the middle Zone of a Spheroid, whose Content in Inches is found by Prop. 9. Sect. 2. and thence in Gall. *&c.* by dividing or multiplying as before.

Exa. Suppose the greatest or Bung Diameter D*d*, 20 Inches, the lesser C*c*, 16, and Height of the Cask AB 31. What is the Content in Ale Gallons?

$$\left.\begin{array}{l} 20\times20=400 \text{ \& } 400\times2=800 \\ 16\times16=256 \end{array}\right\} 1056\times31=32736$$

and 32736 divided by 3.8197, quotes 8570.306 for the Solid Content in Inches, which divided by 282, or multiplied by .003546, gives 30.39 Ale Gallons.

2. If the Staves are not so much curved as before, the Cask is supposed to represent the middle Frustum of a Parabolic Spindle, whose Content in Inches is found by Prop. 14. Sect. 2.

Exa. Suppose (as before) the greatest Diameter 20 Inches, the lesser 16, and Height of the Cask 31 Inches, what is the Content in Ale Gallons?

$20\times20=400$ & $400\times2=800$; then $16\times16=256$ Their Sum is 1056; and $20-16=4$: also $4\times4\times\frac{4}{10}=6.4$ and $1056-6.4=1049.6$, which divided by 3.8197, quotes 274.78, and this multiplied into the Height 31, gives 8518.18 solid Content in Inches, which

which divided by 282, or multiplied by .003546, makes 30.205 Ale Gallons.

3. When the Staves are very little curved, the Cask is ſuppoſed to repreſent the lower Fruſtums of 2 equal Parabolic Conoids, joined together by their greater Baſes at the Bung, whoſe Contents may be found by Prop. 12. Sect. 2.

Exa. Suppoſe (as before) the greateſt Diameter 20 Inches, the leſſer 16, and the Height 31. What is the Content in Ale Gallons?

$20\times20=400$, and $16\times16=256$; their Sum is 656, which multiplied by $9\frac{3}{4}$, produces 30504, and this divided by 3.8197, quotes 7986.17 for the Content in Inches; which divided by 282, or multiplied by .003546, gives 28.31 Gall. of Ale for the Anſwer.

4. Laſtly, if the Staves are pretty ſtreight from the Bung to the Head, as the prick'd Lines in the Figure, the Cask is ſuppoſed to be in Form of the lower Fruſtums of 2 equal Cones joined together by their greateſt Baſes at the Bung, and its Content may be found by Prop. 3. Sect. 2.

$\left.\begin{array}{l}20\times20=400\\16\times16=256\\20\times16=320\end{array}\right\}$ $976\times31=30256$, and this divided by 3.8197, quotes 7921.04 ſolid Inches, which divided by 282, or multiplied by .003546, gives 28.088 Gallons of Ale for the Anſwer.

So the ſeveral Anſwers (according to the ſuppoſed Forms of the Cask) are,

1.	30.39
2.	30.205
3.	28.31
4.	28.088

But ſuch Casks as theſe ought to be reduced to a Cylinder, by taking a mean Diameter, for which obſerve the following Rule.

Multiply the Difference between the Head and Bung Diameter, by .7; .65; .6; or .55, according as the Staves are more or leſs arching, which Product add

to the Head Diameter, the Sum is the mean Diameter, and the Cask is thereby reduced to a Cylinder, whoſe Content is found by Prop. 5. Sect. 3.

Moreover, if you want to know what Quantity of Liquor is drawn out, or remains in any ſuch Cask, when it ſtands upon one of its Baſes;

RULE. As the Square of half the Length of the Cask, is to the Difference between the Head and Bung Areas; ſo is the Square of any Circle's Diſtance from the Bung to the Difference between the Bung Area and that of the Circle, or of the Area of the Liquor's Surface: which found, from the Bung Area take $\frac{1}{3}$ of the foreſaid Difference, and multiply the Remainder by the Liquor's Diſtance from the Bung, and the Product will ſhew what Quantity of Liquor is either above or under $\frac{1}{2}$, the Content of the Cask.

Exa. Suppoſe (as before) the Bung Diameter 20 Inches, the leſſer 16, and Height 31, what is the Content in Ale Gallons, when there is 12 Inches wet?

Half of the Cask's Length is $\frac{31}{2} = 15.5$, whoſe Square is 240.25; The Liquor's Diſtance from the Bung, is $15.5 - 12 = 3.5$, whoſe Square is 12.25. The Area at the Bung is 1.11404 Gall. at the Top .71298, and their Difference is .40106. Therefore

240.25 : .40106 : : 12.25 : .02045, the Difference between the Bung Area and that of the Circle.

Then $1.11404 - .02045 = 1.09359$, which divided by 3, quotes .36453; and $1.11404 - .36453 = .74951$, and this laſt multiplied by 3.5, makes 2.62328 = to what the Cask wants of being half full; wherefore 15.195 (*viz.* half the Content of the whole Cask found by Prop. 14.) $- 2.62328 = 12.57172$, the Number of Ale Gallons in the Cask at 12 Inches wet.

If the Cask had wanted but 12 Inches of being full, the Content would have been $15.195 + 12.57172 = 27.76672$ Gallons.

PROP.

PROP. XV.

To find out what Liquor is in a Cask (not full) when it lies with its Axis parallel to the Horizon.

RULE. By Prop. 9. Sect. 1. find the Area of the Segment in Inches, which reduce to Gallons, *&c.* as is before taught. But becauſe the Area of a Segment may be readily found in Gall. *&c.* by Help of a Table of Segments, as follows: The Quantity of Liquor drawn out or remaining in any Cask lying with its Axis parallel to the Horizon (being firſt reduced to a Cylinder) is beſt found this Way, *viz.* From the Bung Diameter ſubtract the mean Diameter, and half the Difference; alſo from the wet Inches ſubtract the ſaid half Difference, and ſay, As the mean Diameter is to 100 (the Diameter of the Tabular Circle) ſo is the laſt Difference to a vers'd Sine in the Table; then if the Segment ſtanding againſt that vers'd Sine, be multiplied into the Content of the Cask in Gall. or Inches, the Product will ſhew what Quantity of Liquor (in Gall. or Inches) either remains, or is drawn out of the Cask.

Exa. Suppoſe the Cask to repreſent the middle Zone of a Parabolic Spindle, whoſe Bung Diameter is 20 Inches, Head Diameter 16,, Length of the Cask 31, and 8 Inches wet. What is the Content in Ale Gall? Firſt $20 - 16 = 4$, and $4 \times .65 = 2.6$, and $2.6 + 16 = 18.6$ mean Diameter. Then $\frac{20 - 18.6}{2} = .7$, half the Difference between the Bung and mean Diameters, and $8 - .7 = 7.3$ Difference betwixt the wet Inches and the foreſaid half Difference. Therefore

18.6 : 100 :: 7.3 : 39 = .3611 Segment.

The Content of the whole Cask is found by Prop. 5. Sect. 3. to be 29.8685 Gall. wherefore this multiplied by .3611 gives 10.7855 for the Number of wet Gall. and $20 - 8 = 12$, Number of dry Inches; then

12 — .7 = 11.3; therefore 18.6 : 100 : : 11.3 : 60 V. Sine = .6265, which multiplied by 29.8685, gives 18.7126, the Number of dry Gall. the Sum of both is 29.5 = the Content of the whole Cask *fere*.

A TABLE of the Segments of a Circle, whose Area is 1, *the Diameter* (1.128378) *being divided into* 100 *equal Parts.*

V S.	*Segm.*	*V.S.*	*Segm.*	*V.S.*	*Segm.*	*V.S.*	*Segm.*
1	0.0017	26	0.2066	51	0.5127	76	0.8155
2	0.0048	27	0.2178	52	0.5255	77	0.8262
3	0.0087	28	0.2292	53	0.5382	78	0.8369
4	0.0134	29	0.2407	54	0.5509	79	0.8474
5	0.0187	30	0.2523	55	0.5635	80	0.8576
6	0.0245	31	0.2640	56	0.5762	81	0.8677
7	0.0308	32	0.2759	57	0.5888	82	0.8776
8	0.0375	33	0.2878	58	0.6014	83	0.8873
9	0.0446	34	0.2998	59	0.6140	84	0.8968
10	0.0520	35	0.3119	60	0.6265	85	0.9059
11	0.0598	36	0.3241	61	0.6389	86	0.9149
12	0.0680	37	0.3364	62	0.6514	87	0.9236
13	0.0764	38	0.3486	63	0.6636	88	0.9320
14	0.0851	39	0.3611	64	0.6759	89	0.9402
15	0.0941	40	0.3735	65	0.6881	90	0.9480
16	0.1032	41	0.3860	66	0.7002	91	0.9554
17	0.1127	42	0.3986	67	0.7122	92	0.9625
18	0.1224	43	0.4112	68	0.7241	93	0.9692
19	0.1323	44	0.4238	69	0.7360	94	0.9755
20	0.1424	45	0.4365	70	0.7477	95	0.9813
21	0.1526	46	0.4491	71	0.7593	96	0.9866
22	0.1631	47	0.4618	72	0.7708	97	0.9913
23	0.1738	48	0.4745	73	0.7822	98	0.9952
24	0.1845	49	0.4873	74	0.7934	99	0.9983
25	0.1955	50	0.5000	75	0.8045	100	1.0000

Of MALT-GAUGING.

According to an Act of an *Engliſh* Parliament *anno* 1697, every round Buſhel with a plain and even Bottom, 18 ½ Inches wide throughout; and 8 Inches deep, ſhould be eſteem'd a legal *Wincheſter* Buſhel: Now ſuch a Veſſel will contain 2150.42 Cubic Inches; for 18.5×15.5 = 342.25, which multiplied by .7854, gives 268.80315, and this laſt multiplied by the Height 8, produces 2150.42. And therefore to find the Number of Buſhels contained in any Veſſel, firſt find its Solidity in Inches, according to the Form of the Veſſel, and divide by 2150.42 for the Anſwer.

If the Malt be lying on the Floor, in order to know the true Depth, you muſt take the Depth in ſeveral (ſuppoſe 6, 7, 8, or more) Places, the Sum of which divided by the Number of Places you took the Depth in, quotes the mean Depth.

Exa. Suppoſe a Quantity of Malt lying on the Floor, in Form of a Rectangular Parallelogram, Length 160 Inches, and Breadth 100 Inches, what is the Number of Buſhels contain'd in it?

Suppoſe	1	5.5
	2	6.
	3	4.8
	Depth to be	
	4	5.9
	5	4.8
	6	6.1
	7	5.7
		38.8

the Sum is 38.8, which divided by 7, the Number of Places, the Quote 5.543 is the mean Depth. Then 160×100 = 16000, and 16000×5.543 produces 88688 Cubic Inches, which divided by 2150.42, quotes 41.242 Buſhels for the Anſwer.

FINIS.

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